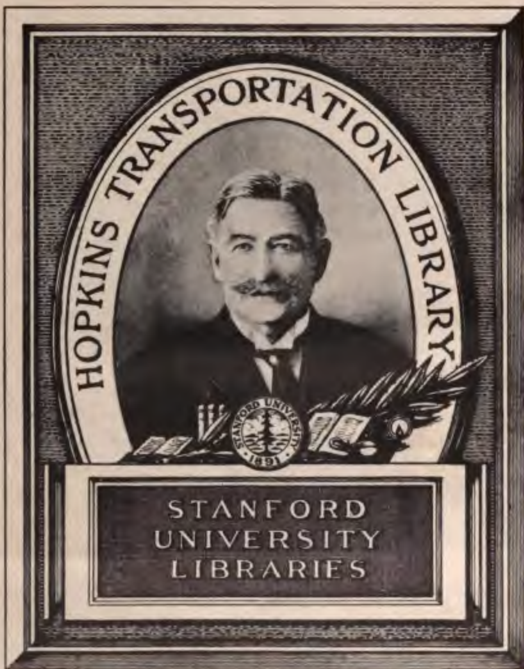

HOPKINS TRANSPORTATION LIBRARY
STANFORD UNIVERSITY 1891
STANFORD
UNIVERSITY
LIBRARIES

CONTENTS.

SECTION VII.—ON THE USE OF THE OBLIQUE-ANGLED TRIANGLE

SECTION VIII.—ON THE SECTION OF THE CYLINDER.

SECTION IX.—ON THE ANGLE OF THE TWIST.

SECTION X.—ON THE TRIHEDRAL.

SECTION XI.—OBLIQUE ARCH.

PART II.

TREATISE ON THE OBLIQUE ARCH WITH SPIRAL JOINTS.

PART III.

THEORY OF THE OBLIQUE ARCH.

DIRECTIONS TO THE BINDER.

INTRODUCTION.

Observe that Plate lv. is by mistake marked vl.

OBLIQUE ARCH WITH SPIRAL JOINTS.

PREFACE.

In this undertaking, the general reader is not supposed to be much acquainted with scientific researches, but to have some knowledge of descriptive geometry: the author, therefore, before he could venture to explain the principles of the oblique arch, and to reduce them to practice, has been induced to lay before his readers those problems which would afterwards be required, and that they may always be ready for the use of the working mason, or inspector, without being under the necessity of having recourse to other publications. In this treatise, the geometrical constructions and the calculations are quite independent of each other; each system is complete of itself: so that the reader who is not acquainted with the principles of algebra may pass the formulae and proceed with the explanation of the figures and diagrams; but such as are able to understand the literal expressions will soon find their utility and the great convenience and facility which they afford in the construction of the oblique arch, for in making working drawings of developements to the full size, so much room is required to lay them down, that it is often difficult to find a place which will contain them; the advantages, therefore, of calculation, are obvious. In this treatise, every useful length, or distance, or angle, of an oblique arch, has been found principally by common arithmetic, from the doctrine of similar triangles. The principles of calculation have been applied to three oblique arches, which have been executed.

In the author's treatise on stone-cutting, the construction of the oblique arch is given by a reverse process to that employed in the present work: it was there considered as necessary first to form the cylindric surface, from which the spiral surfaces of the beds might be more easily and more exactly obtained; stone cutters are, however, generally inclined to regulate the face from the bed, and, upon consideration, that as the bed might be wrought by straight edges, the reverse process gives greater facility in the execution of the work, but whether we commence working the arch-stones with the soffit or the bed, the same templets would be required. Previously, however, to working the bed, it will be necessary to ascertain the angle of the twist. The forms of the templets are exhibited at No. 1, No. 2, No. 3, No. 4 (Plate 26); they are not shewn by any other author who has written upon the oblique arch. Of these templets, No. 3 and No. 4, called arch squares, are employed for the squaring the arch stones; No. 3 is used in forming the beds and soffits, viz.: having wrought one bed, by means of the winding rules, No. 5, No. 6, the position and form of the cylindric surface of the soffit may be ascertained, by applying No. 3 in the same manner as a common square, and having finished one bed and the soffit, the position and form of the spiral surface of the remaining bed may also be ascertained by No. 3 The other arch square, No. 4, is employed in forming the end of the stone, which is also a spiral surface. In the application of these arch squares, the curved edge must rest upon the cylindric surface of the soffit, and thus the three faces of every arch stone may be determined. Here it may be observed that as the workman is not restricted, he may woık the beds at one operation of any length which he may find convenient. Every other means besides this of forming the arch-stones will be liable to great inaccuracy. Though the instrument, first described in the author's treatise on stone-cutting, is derived from a correct principle, it is difficult to keep it in a steady position upon the stone, yet this is the instrument which is used in other publications for squaring the stones.

It is very inconvenient to lay out the whole developement of an oblique arch to the full size; but it is only necessary to find the devolopement of half the intradosal line; for by making a mould to the half thus found, the other half may be drawn by reversing the ends of the mould and placing the curve on the other side of the line of subtense, and thus we shall have the entire curve,—see Plate A.

It has become fashionable for writers on subjects even connected with science to use no definitions, the author is however of a different opinion, he has taken every opportunity to explain the complex parts of the oblique arch which could not be done by simple definitions. It will, therefore, be of great advantage to the reader to peruse with attention the descriptive definitions, beginning page 38, before he begins to study the work itself.

Considering the great expense of the tables which are necessary for making the calculations, and the number of accurate figures represented in the plates, the author flatters himself that this will not only be found a cheap, but a useful publication, to all who are desirous of acquiring a thorough knowledge of the principles of the oblique arch.

In order that a book with plates may be easily consulted, so as to save the reader the least possible trouble, every plate should be placed opposite to the page of letter press which contains its explanation, and this can always be done provided the type is sufficiently small. In this work the size of the type has been so chosen, that out of 40 plates, 28 have been explained in the opposite pages. The plates which are not opposite to the letter press, being 12 in number, belong chiefly to the introduction.

Most of the oblique arches in this part of the country have been constructed according to the principles shown in this treatise, particularly those upon the Newcastle and North Shields

[illegible] was therefore anxious to ascertain how [illegible] what difficulties the workmen had found in [illegible] applied to the engineer upon the line, he [illegible] the following report :—

'Royal Arcade, Newcastle upon Tyne,
'29th July, 1839.

[illegible] to your enquiry, I beg to state that I have built [illegible] the Newcastle and North Shields Railway. [illegible] have stood extremely well. The [illegible] in dressing or setting the stones correctly, after [illegible]

'I remain, dear Sir,
'Your's most truly,
'ROBT. NICHOLSON.

[illegible] Esq., Newcastle upon Tyne."

A

TREATISE

ON

THE OBLIQUE ARCH.

INTRODUCTION.

SECTION I.

GEOMETRY.

The meaning of a right angle, an acute angle, an obtuse angle, and of a right-angled triangle, an acute-angled triangle, and an obtuse-angled triangle, is supposed to be understood either from the reader's own practice or otherwise; more than this, however, is in general, but very imperfectly known. As angles so very frequently require to be found under various circumstances, the following short discussion upon their properties, their construction, and the construction of right-angled triangles, will render the knowledge of them clear and familiar to the understanding of the workman for whose use this Treatise is principally intended.

ON THE NATURE OF ANGLES.

If from the point in which the two straight lines forming a right angle meet each other, the arc of a circle be described to meet each line, and if the arc be divided into ninety equal parts, and straight lines be drawn from the centre through each point of division, the right angle will be divided into ninety equal angles, each of which is called a *degree;* and if each degree be again divided into sixty equal parts, and straight lines be drawn to the centre, as before, each of these small angles is called a *minute.*

A number having a small zero or cypher placed over the right hand shoulder of the figure, or last figure, shows this number to be as many degrees as the figure or figures express, and an accent placed in the same manner over a number, shows this number to be as many minutes as the figure or figures express. Thus $36° 23'$ mean thirty-six degrees, twenty-three minutes.

[illegible] meeting of two lines, which contain an angle, is [illegible] *of the angle.*

[illegible] by the arc of a circle, described from the [illegible] containing the angle.

[illegible] IN THE CONSTRUCTION OF ANGLES.

[illegible] the two ends of an arc is called the chord of [illegible] *chord.*

[illegible] the ends of an arc, taken with a pair of com- [illegible] *extent* or *subtense* of the arc.

[illegible] drawn from the centre to meet the chord of an arc [illegible] will divide the chord into two equal parts, and if the [illegible] produced to meet the arc, the arc will also be di- [illegible] equal parts.

[illegible] the chord is called the sine of the half arc to which it [illegible]

[illegible] drawn from the centre, to meet the chord perpendicularly, [illegible] *sine of the half arc.*

[illegible] radius, the sine, and the co-sine form a right-angled [illegible] which the sine is opposite to the angle subtended by the [illegible]

PRINCIPLE OF CONSTRUCTING ANGLES.

[illegible] chord of 60 is equal to the radius:

[illegible] any straight line as a diameter describe a semi-circle. Di- [illegible] into three equal parts, and draw the two intermediate [illegible] the chords of the three arcs; then the figure will be divided [illegible] isosceles triangles of which each angle at the centre is one [illegible] two right angles; but if an isosceles triangle have the angle [illegible] by the equal sides equal to one-third of two right angles, [illegible] is equilateral; hence each of the two equal sides, is equal [illegible] third, but the two equal sides of each triangle are radii, and [illegible] side is the chord of one of the equal arcs; therefore the [illegible] equal to the chord; but the angle at the centre being one- [illegible] two right angles, is one third of 180° equal to 60°; hence the [illegible] is equal to the radius.

ON THE MEASUREMENT OF ANGLES.

[illegible] ratio of the radius and the chord of one arc being equal to the [illegible] the radius and the chord of another arc, the angle contained [illegible] two radii drawn from the extremities of the one arc is equal to [illegible] angle contained by the two radii drawn from the extremities of [illegible] arc. Therefore if an arc be described with a radius equal [illegible] chord of 60°, and if a given number of degrees taken from a [illegible] chords, and the extent placed upon the arc, and if through [illegible] extremity a line be drawn to the centre, the two radii will con- [illegible] angle of as many degrees, the radius being equal to the chord [illegible]; if the extent set upon the arc be the chord of 90°, the angle [illegible] contained by the two radii will be a right angle.

TO CONSTRUCT A SCALE OF CHORDS.

Draw a straight line *A B* (Fig. I.) and from *B* any point in *A B* draw *B C* perpendicular to *A B*. Make *B A* equal to the given radius, and from *B* with the distance *B A* describe the arc *A C*. From *C* with the distance *A B* cut the arc *A C* in *d*, and from *A* with the same radius, cut the arc *A C* in *e*; then the quadrant *A C* is divided into three equal parts at the points *d* and *e*. Divide *A d*, *d e*, *e C* each also into three equal parts, and the whole arc *A C* will thus be divided into nine equal parts. Divide each of these nine into ten equal parts, and the whole arc *A C* will be divided into ninety equal parts. Draw the chord *A C*, and from *A* with the distance of each point of division in the arc *A d e C*, cut the straight line *A C* as shown by the numbers 10, 20, 30, &c., and *A C* is a scale of chords.

Figure 2 is a scale of chords transferred from figure 1, and is that which is referred to in the following constructions and in the mensuration of angles.

Figure 3 is a scale of equal parts as used in the construction of triangles, and is the scale referred to. The parts may represent feet, yards, chains, &c.

CONSTRUCTING OF ANGLES.

From a given point *A*, Fig. 4, upon a given straight line *A B*, to construct a right angle by the scale of chords.

From *A*, with a chord of 60°, describe an arc *B C*; make *B C* equal to the chord of 90°; draw *A C*, and the angle *B A C* shall be a right angle, or *A C* shall be perpendicular to *A B*.

From a given point as a vertex, upon a given straight line to construct an angle, which shall contain a given number of degrees, &c.

From the vertex, with the subtense of 60° taken from the scale of chords, describe an arc meeting the given line; from the point of meeting with a distance equal to the chord of the given angle, cut the arc; draw from the vertex a straight line through the point of intersection, and the two lines shall contain the angle required.

EXAMPLE.

At a given point *A*, Fig. 5, upon a given straight line *A B*, construct an angle of 35°.

From *A* with a chord of 60°, describe the arc *d e* meeting *A B* in *d*, from *d*, with the chord of 35°, cut the arc *d e* in *e*; through *e* draw *A C*, and the angle *B A C* shall contain 35°.

To make an angle equal to any number of degrees greater than 90° but less than 180°.

[The student is requested to supply the diagram for this proposition, it being omitted for want of room.]

Let *A* be a given vertex, and *A B* one of the given lines containing the angle; from *A*, with the chord of 60°, describe an arc *B C*; from the point *B*, with thechord of 90°, cut the arc *B C* in *d*; make *d C*

from the scale of chords equal to the excess of the given number of degrees above 90° ; draw *A C*, and *B A C* is the angle required.

An Angle *B A C*, Fig. 5, being given to find the number of degrees it contains.

From *A*, with the chord of 60°, describe an arc *d e*, meeting *A B* in *d*, and *A C* in *e*; apply the distance *d e* to the scale of chords, and the point of extension will show the number of degrees. This will he found to be nearly 35°.

To make an angle *B A C*, Fig. 6 of 60°, without using the scale of chords.

From *A*, with any radius, describe the arc *B C*; from *B*, with the same radius, cut the arc *B C* in *C*; join *A C*, and *B A C*, is the angle required.

For if a straight line *B C* be drawn, *A B C* will be an equilateral triangle of which the angle *B A C* will be 60°, as well as the angles at *B* and *C*.

To make an angle *B A C*, Fig. 7, of 30°, without using the scale of chords.

Take from *A* upon *A B*, the two equal distances *A d*, *d e ;* from *d* with the distance *d e* describe the arc *e f;* from *e* with the same distance cut the arc *e f* in *f;* through *f* draw *A C*, and *B A C*, is the angle required.

For if the chord *e f* and the radius *d f* be drawn, *d e f* will be an equilateral triangle, and if the semicircle *A f e* be completed, the angle *e d f* at the centre will be double the angle *e A f* or *B A C* at the circumference ; but *e d f* being an equiletral triangle, the angle *e d f* is 60° ; therefore the angle *B A C*, is 30°.

OF THE COMPLEMENT AND SUPPLEMENT OF AN ANGLE.

The difference between any angle and a right angle is called the complement of that angle.

Thus 55° the difference between 35° and 90° is called the complement of 35°, and reciprocally 35° the difference between 55° and 90° is the complement of 55°.

The difference between an angle and two right angles, is called the supplement of that angle.

Thus 143° the difference between 37°, and 180° is called the supplement of 37°, and reciprocally 37° the difference between 143° and 180° is the supplement of 143°.

TRIANGLES.

The angles, as well as the sides of a triangle, are called *parts*, which are, therefore, six in number, of which any three being given, except the three angles, the other three may be found. In a right-angled triangle, as the right angle is always given, any two of the remaining parts, except the two angles, will be sufficient to construct the triangle ; and thus, in respect of the given data, we have five cases of right-angled triangles, which are as follows :—

Two parts being given to construct a right-angled triangle.

CASE 1.

Given the two sides, which contain the right angle, to construct the triangle.

From the same point draw two straight lines perpendicular to each other; make these lines respectively equal to the two sides; join the unconnected extremities, and the figure will be the triangle required.

EXAMPLE.

Construct a right angled triangle, of which one side shall be 35 and the other 19 feet.

Draw *A B*, Fig. 8, and *B C* perpendicular to *A B*; make *A B* equal to 35 feet, and *B C* equal to 19 feet; join *A C*, and *A B C* is the triangle required.

CASE 2.

Given the hypothenuse and one of the other two sides, to construct the triangle.

Draw a straight line, and make it equal to the hypothenuse. Upon the hypothenuse, as a diameter, describe a semicircle; from one end of the diameter, with the length of the other side, cut the semicircular arc; join the point of intersection and each end of the diameter, and the figure is the triangle required.

EXAMPLE.

The hypothenuse being 40 feet, and the other side 35, construct the triangle.

Draw *A C* Fig. 9; make *A C* equal to 40 feet; upon *A C*, as a diameter, describe the semicircle *A B C;* from *A*, with the length 35 feet of the other side, cut the semicircular arc in *B;* join *B A*, *B C*, and the figure *A B C* is the triangle required.

CASE 3.

Given one of the two sides about the right angle, and the angle adjacent to that side to construct the triangle.

Draw a straight line; make it equal to the given length; from one end raise a perpendicular; at the other end of the given line make an angle equal to the given angle, and the figure made by the meeting of the lines shall be the triangle required.

EXAMPLE.

One side about the right angle being 35 feet, and the adjacent angle 28° 30′ construct the triangle.

Draw *A B*, Fig. 8; make *A B* equal to 35 feet; draw *B C* perpen-pendicular to *A B;* make the angle *B A C* equal to 28° 30′, and the figure *A B C* is the triangle required.

CASE 4.

... the two sides about the right angle, and the opposite angle to that side to construct the triangle.

... the complement of the given angle, and we shall have an angle adjacent to the given side; therefore, proceed as in case 3, and the ... constructed shall be the triangle required.

EXAMPLE.

... one side about the right angle equal to 35 feet, and the opposite angle equal to 61° 30′ construct the triangle.

The compliment of 61° 30′, is 28° 30′; proceed now as in the example to case 3, and this shall be the triangle required.

CASE 5.

Given the hypothenuse and an angle to construct the triangle.

Draw a straight line; make it equal to the hypothenuse; upon the given line as a diameter, describe a semicircle; make an angle with the diameter at one of its ends equal to the given angle; draw a straight line from the other extremity to the point of intersection, and the figure shall be the triangle required.

EXAMPLE.

Given the hypothenuse equal to 40 feet, and one of the angles 28° 30′, construct the triangle.

Draw *A C*, Fig. 9; make *A C* equal to 40 feet; upon *A C* describe the semicircular arc *A B C*; make the angle *C A B* equal to 28° 30′; draw *B C*, and the figure *A B C* is the triangle required.

In any of the five cases to find the parts of the triangle required.— If angles they may be ascertained from the scale of chords, and if sides from the scale of equal parts.

THE NAMES OF THE THREE SIDES OF A RIGHT-ANGLED TRIANGLE.

If the hypothenuse be called radius, the side opposite to either of the acute angles is called *the sine* of that angle, and the remaining side, which, with the radius contains the angle, is called *the cosine* of the angle. *See* page 2. Thus in the triangle *A B C*, Fig. 10, if *A C* be called radius, the side *B C* opposite the angle *A* is called the *sine*, and side *A B* the co-sine of the angle *A*.

If one of the sides about the right angle be called radius, the other side is called *the tangent* of the opposite angle, and the hypothenuse *the secant* of the same angle. Thus in the right angled triangle *A B C*, Fig 11, if *A B* be called the radius, *B C* is called the tangent of the angle *A*, and *A C* the secant of the same angle *A*.

Given the three parts of an oblique-angled triangle to construct the triangle.

CASE 1.

Given the three sides of which any two is greater than the third to construct the triangle.

Draw a straight line, make it equal in length to one of the sides, from one end of the line with the length of another side describe an arc; from the other end describe another arc to intersect the other; join the point of intersection, and each end of the line, and the figure shall be the triangle required.

EXAMPLE.

Given the three sides respectively equal to 78, 57, and 38 feet construct the triangle.

Draw *A B* Fig. 1.; make *A B* equal to 78 feet; from *A* with the distance of 57 feet, describe an arc; from *B* with the distance of 38 feet, describe another arc, intersecting the former at *C*; join *C A*, *C B*, and the figure *A B C*, is the triangle required.

CASE 2.

Given the two sides and the contained angle to construct the triangle.

Draw a straight line; make the line equal in length to one of the given sides; from one end of the line make an angle equal to the given angle; from the vertex upon the unlimited line, set off the length of one of the other two sides; join the unconnected ends of the two lines, and the figure shall be the triangle required.

EXAMPLE.

Given the two sides respectively equal to 78 and 57 feet, and the contained angle 27° 30′ to construct the triangle.

Draw *A B* Fig. 1.; make *A B* equal to 78 feet; make the angle *B A C*, equal to 27° 30′; make *A C* equal to 57 feet; join *B C*, and *A B C* is the triangle required.

CASE 3.

Given two sides, and an angle opposite to one of them, to construct the triangle.

Draw a straight line; make the length of the line equal to one of the given sides; at one end of the line make an angle equal to the given angle; from the other end, with the length of the other side, cut the unlimited side of the given angle; join the end of the line and the point of intersection, and the figure shall be the triangle required.

EXAMPLE.

Given two sides of a triangle respectively equal to 78 and 57 feet, and the angle opposite to the side 57 equal to 43° 30′, to construct the triangle.

Draw $A\ B$, Fig. 1; make $A\ B$ equal to 78 feet; make the angle $A\ B\ C$ equal to 43° 30′; from A, with the distance of 57 feet, cut $B\ C$ in C; join $A\ C$, and $A\ B\ C$ is the triangle required.

Case 4.

Given a side, and the two adjacent angles to construct the triangle.

Draw a straight line; make the line equal to the length of the given side; at each end of the line make angles respectively equal to the two given angles, and the figure shall be the triangle required·

Example.

Given one side equal to 78 feet, and the two angles respectively equal to 26°, and 43° 30′ constrnct the triangle.

Draw $A\ B$, Fig. 1; make $A\ B$ equal to 78 feet; make the angle $B\ A\ C$ equal to 27° 30′; the angle $A\ B\ C$ equal to 43° 30′, and the figure $A\ B\ C$ is the triangle required.

Case 5.

Given two angles and a side opposite to one of them to construct the triangle.

In this construction the line which is to be opposite to one of the given angles will be adjacent to the other.

Draw a straight line; make the line equal to the length of the given side; make an angle at one end of the line equal to the adjacent angle; make an angle at the other end of the line equal to the supplement of the sum of the two given angles, and the figure enclosed by the three lines is the triangle required.

Example.

Given the two angles respectively equal to 27° 30′, and the side opposite to 43° 30′ equal to 57 feet, construct the triangle.

Draw $A\ C$ (Fig. 1); make $A\ C$ equal to 57 feet; make the angle $C\ A\ B$ equal to 27° 30′; make the angle $A\ C\ B$ equal to 109°, which is the supplement of 71°, the sum of 27° 30′, and 43° 30′, and the figure enclosed by the three straight lines is the triangle required.

In any case when a triangle is constructed to find the remaining parts; the length of the sides will be found by applying the lengths found by construction upon the scale of equal parts, and the measurement of angles by the scale of chords.

BISECTION OF A LINE PERPENDICULARLY.

To bisect a given straight line $A\ B$, Fig. 2, that is to divide it into two equal parts.

From A, with any distance greater than the half of $A\ B$, describe an arc; from B, with the same radius, describe another arc intersecting the former arc at d and e; through the points d, e, draw the straight line $F\ G$, and $F\ G$ will bisect $A\ B$ perpendicularly in C.

PROBLEMS REQUIRED IN PRACTICE.

PROB. I.—From a given point C (Fig. 3), near the middle of a straight line $A\ B$ to draw a perpendicular.

On each side of the point C, upon the line $A\ B$, take two equal distances $C\ e$, $C\ f$; with any radius greater than $C\ e$ or $C\ f$ describe an arc from the point e; with the same radius describe another arc from the point f intersecting the former arc in g; through g draw the straight line $C\ D$, and $C\ D$ is perpendicular to $A\ B$.

PROB. II.—From a given point B (Fig. 4), at the end of a given straight line $A\ B$, to draw a perpendicular.

Make $A\ B$, taken from a scale of equal parts, equal to 4 feet; from B, with a distance equal to 3 feet, describe an arc; from A, with a distance equal to 5 feet, describe another arc intersecting the former in C; join $B\ C$, and $B\ C$ is perpendicular to $A\ B$.*

The same thing may be done by using the numbers 6, 8, 10, instead of 3, 4, 5, or any numbers in the proportion of 3, 4, 5.

PROB. III.—To bisect a given angle $A\ B\ C$, Fig. 5.

From B, with any radius, describe an arc $d\ e$, meeting $B\ A$ in d, and $B\ C$ in e; from d, with any radius greater than half the distance between d and e, describe an arc; from e, with the same distance, describe another arc, meeting the former in f; through f draw $B\ C$, and $B\ C$ will bisect or divide the angle $A\ B\ C$ into two equal angles.

PROB. IV.—To make an angle at the point E (Fig. 6), with the straight line $D\ E$, that shall contain an angle equal to the given angle $A\ B\ C$.

From B, with any convenient radius, describe the arc $d\ e$, meeting $B\ A$ in d, and $B\ C$ in e; from E, with the same radius, describe an arc $g\ h$, meeting $E\ D$ in g; make $g\ h$ equal to $d\ e$; through h draw the straight line $E\ F$, and $D\ E\ F$ is the angle required.

PROB. V.—To make a straight line equal to the length of a given arc.

The thing here proposed to be done cannot be effected by any rule founded upon geometrical principles. It is evident that if a given arc be divided into equal parts, and the chords of these arcs be as often repeated upon a straight line, the multiple thus extended will be less than the length of the arc; the smaller, however, the dis-

* For if $A\ C$ be drawn, and if the square of $A\ C$ be equal to the sum of the squares of $A\ B$, $B\ C$, the triangle $A\ B\ C$ shall be a right angled triangle (Euclid, Book 1, Proposition 48), the right angle being at B; therefore, because $(n^2+1)^2=(n^2-1)^2+(2\ n)^2$, the numbers n^2+1, n^2-1, $2\ n$, are the sides of a right angled triangle; thus let $n=2$, then $n^2+1=5$, $n^2-1=3$, $2\ n=4$; therefore because the sides of the triangle $A\ B\ C$ are 5, 4, 3, it is right angled. The numbers 3, 4, 5, or their doubles, 6, 8, 10, are very convenient for the use of workmen; there are numbers, however, in a very different proportion that will answer the same purpose; thus let $n=4$, then $n^2+1=17$, $n^2-1=15$, $2\ n=8$; hence 17, 15, 8, are the sides of a right angled triangle, but more inconvenient than the others.

tance is between the points, and the more numerous the parts are, the straight line which contains the multiple shall be the more nearly equal to the length of the arc.

Example.

Find the length of the arc *A B*, Fig. 7.

Take a small distance between the points of the compasses, and suppose the arc to contain this distance eight times from *A* to *c* with a remainder *c B*; draw the straight line *D E*, upon which repeat the chord as often as the nnmber of small equal arcs are contained upon the whole arc from *D* to *f*; make *f E* equal to the remaining part *e B* of the arc, then will *D E* be nearly equal to the arc *A B*.

Observation.

In practical works, when the points through which a curve required to be drawn are given by being previously found, the curve is drawn by placing small nails in the points, and bending a thin slip of wood, or other elastic substance, round the points, and by drawing a curve by the side of the slip next to the nails; then the length of the slip extended in a straight line will be equal very nearly to the length of the arc or curve. The length of the curve obtained in this way is perhaps as exact as it is possible to be found.

Prob. VI.—Upon a straight line *A C*, Fig. 8 as a chord to describe the arc of a circle, the height of the middle of the arc being given.

Bisect *A C* by a perpendicular *B E* intersecting *A C* in *D*; make *D E* equal to the height of the arc; draw the chord *A B* of the half arc; bisect *A B* by a perpendicular, meeting *B E* in *E*; from *E*, with the distance *E B*, describe the arc *A B C*, and *A B C* is the arc required.

It frequently happens that the distance of the centre of the circle is so very great as to make it inconvenient to get a radius and the portion of the arc of the circle required so very small, recourse must be had to the method of describing an arc of a circle without using the centre by means of an angle.

Prob. VII.—The same things being given to describe the arc without making use of the centre.

Draw the straight line *A C*, Fig. 9, and make *A C* equal to the chord; bisect *A C* by a perpendicular *B D*, meeting *A C* in *D*; make *D B* equal to the height of the arc. Having prepared two thin slips of wood with straight edges, each being a little longer than the chord *A C*; lay the one slip upon the other so that the straight edges being outermost, may intersect each other at *B*; one of the straight edges resting upon a pin at *A*, and the other upon a pin at *C*; fix these two slips together at *B*, and to keep them invariably to the angle, fasten another slip *G H*, at each end to each of the other two; move the angle *E B F*, so that the side *B E* may slide upon the point *A*, and the side *B F* upon the point *C*, while a pencil being held to the vertex *B* of the angle, will describe the arc *A B C*.

Another method still more convenient.

Let A C (Fig. 1) be a straight line equal in length to the chord. Bisect A C in g, and draw g h perpendicular to A C, and make g h equal to the height of the arc. Draw h D parallel to A C, and join h A, which prolong to any convenient point E. Make h D equal to h E, and join E D. Let A, h, C, Fig. 2 be points at the same distances from one another as the points A, h, C, Fig. 1. In Fig. 2 fix two pins one in A and the other in h, and move the angle E B D so that the edge B E may slide upon the pin A and the edge B D upon the pin h while during the motion a pencil being held at the point B of the angle E B D will describe the arc A h. In the same manner, by taking the pin out of A and placing it in C, the arc h C will be described which will complete the entire arc A h C of which the chord is A C and the height g h.

PROB. VIII.—The two axes of an ellipse being given to find any number of points in the curve.

Let A B (Fig. 3) be the axis major. Bisect A B in C by the perpendicular E D. Make C E equal to the semi-axis minor, and make C D equal to C E. Through E draw F G parallel to A B, and draw A F and B G parallel to D E. Divide A F, A G, each into the same number of equal parts at 1, 2, 3, &c. From the points 1, 2, 3, &c., in A F draw 1 E, 2 E, 3 E, &c., and from D through the points 1, 2, 3, &c , in A C, draw D b, D c, D d, &c., meeting the lines 1 E, 2 E, 3 E, &c., in the points b, c, d, &c. In the same manner find the points b, c, d, &c., on the other side of D E. From the point A and through the points b, c, d, &c., draw the curve A b c d E, and in the same manner draw the curve B b c d E.

PROB. IX.—Given the two axes of an ellipse, to find the radius of curvature at the extremities of the axes, thence to describe an approximate curve by the arcs of circles.

Let A B (Fig. 4) be the axis major. Bisect A B in C by the perpendicular E D, and make C E, equal to the semi-axis minor. In (Fig. 4. No. 2) draw i p and i n at any convenient angle. In i p make i m, i k respectively equal to C A, C E (Fig. 4, No. 1). From the point i (Fig. 4, No. 2) describe the arcs m n, k l meeting i n in n and l, and join m l. Draw n p parallel to l m, and draw k q also parallel to l m meeting i n in q. Make E D No. 1, equal to i p No. 2, and upon A B. No. 1, make A e, B f, each equal to i q No, 2. From the centre D No. 1, with the distance D E describe the arc G H, and from the centres e, f with the distance A e equal to B f describe the arcs A L, B K. Then the three arcs A L, B K, G H, will very nearly coincide with the curvature of an ellipse drawn upon correct principles. The remaining portions of the curve may be traced by hand. In order to find another point in each curve between the arc thus described ; from E the extremity of the semi-axis minor, with the distance C A or C B, the semi-axis major, cut A C at m, and B C at n. Parallel to C E draw m p, n q ; make m p, n q each equal to A e or B f, and the points p and q are in the curve.

Observation.

The radius of curvature at the extremity of the semi-axis minor, and at each extremity of the semi-axis major, may be found by calculalation as shown in the observations after Problem XIII.

The points *m*, *n*, are the two focii of the semi-ellipse *A E B*, and the lines *m p*, *n q*; the ordinates to the curve passing through the focii are each termed the latus rectum, which is a third proportional to the semi-axis major and the semi-axis minor as shown by the writers on conic sections.

THE USE OF THE TRAMMEL.

The trammel consists of two parts, of which one is fixed and the other moveable; the fixed part is a plate of metal or a thin piece of wood of inconsiderable thickness, comprised between parallel plane surfaces, with two grooves receding from one of the faces at right angles to each other, and the moveable part is a bar having a hole through it at one end to hold a pencil, and two pieces made to slide along its sides with a steel pin in each slider; these pins are of a cylindrical form, so that the centres of their circular ends, and the point of the pencil may always be in a straight line, and to admit of being fixed at any distances from the pencil point, and that the cylindric pins may be fitted exactly, and to move freely in the grooves.

Prob. X.—The axes of an ellipse being given to describe the curve by a trammel.

Place the middle of the groove *G H* (Fig. 5) upon the axis major *A B*, so that the intersection may fall upon the centre *C*. Let *i p* be the edge of the trammel bar, and *p* the point of the pencil. Slide each of the moveable pieces, so that their distances from *p* may be respectively equal to *C E*, *C A*. Then place *k* the centre of the nearest steel point in the groove *G H*, and *i* the centre of the most remote in the groove *F C*. Move the end *p* of the bar while the points *k*, *i*, remain in the grooves, and the pencil at *p* will describe the curve.

Prob. XI.—Given the semi-axis minor, the abscissa upon the semi-axis minor an ordinate parallel to the axis major, to describe the curve.

Let *e E* (Fig. 6) be the semi-axis minor, *e* the centre, and *h A* or *h B* an ordinate. Prolong *E e* to *m*, and through *e* draw *C D* parallel to *A B*. From *B* with the distance *E e* cut *e D* in *f*; join *B f* and prolong *B f* to meet *e m* in *q*. In *e C* take *e k* less than *e f*, and from the point *k* with the distance *f q* cut *e m* in *i*, join *i k* and prolong *i k* to *p*. Make *k p* equal to *e E* or *f B*, and *p* is the point in the ellipse. In the same manner may be found as many points as will be sufficient to draw the curve, but which may be better drawn by the trammel.

Prob. XII.—To describe a curve comprised by five circular arcs which shall be a near approximation to the curve of a given semi-ellipse.

Let $A\ G$ (Fig. 1) be the axis major. Bisect $A\ G$ in M by the perpendicular $D\ J$, and make $M\ D$ equal to the semi-axis minor. Draw $D\ p$ making any given angle with $D\ J$. From D with the distance $D\ M$ cut $D\ p$ in n, and again from D with the distance $A\ M$ or $M\ G$, cut $D\ J$ in q, and $D\ p$ in p. Join $q\ n$ and parallel to $q\ n$, draw $p\ J$ and draw $M\ r$ parallel to $q\ n$, meeting $D\ p$ in r. From J with the radius $J\ D$ describe the arc $C\ E$ so that $D\ C$ and $D\ E$ may be each about 15° or $C\ E$ about 30° and join $C\ J$ and $E\ J$. In $A\ G$ make $A\ H$ and $G\ L$ each equal to $D\ r$. In $C\ J$. make $C\ s$ equal to $A\ H$, and join $H\ s$. Bisect $H\ s$ by a perpendicular $t\ I$ meeting $C\ J$ in I. Join $I\ H$ and prolong $I\ H$ to B. In $J\ E$, make $J\ K$ equal to $J\ I$. Join $K\ L$, and prolong $K\ L$ to F. From the centre I, with the distance $I\ C$, describe the arc $B\ C$, and from the centre H, with the distance $H\ B$ describe the arc $A\ B$. Also from the centre K with the distance $K\ E$, describe the arc $E\ F$, and from the centre L with the distance $L\ F$, describe the arc $F\ G$; then the curve $A\ B\ C\ D\ E\ F\ G$ will be a very near approximation to the curve of a semi-ellipse.

Prob. XIII.—To describe a curve composed by circular arcs from seven centres which shall be a very near approximation to the curve of a semi-ellipse, the two axes being given.

Let $A\ I$ (Fig. 2) be the axis major. Bisect $A\ I$ in J by a perpendicular $E\ N$, and make $J\ E$ equal to the semi-axis minor. By Problem VIII., find the points B, C, D. Bisect $D\ E$ by a perpendicular $t\ N$, and join $D\ N$, and suppose $C\ D$ joined. Bisect $C\ D$ by a perpendicular $u\ M$, meeting $D\ N$ in M, and join $C\ M$. Suppose $B\ C$ joined, and bisect $B\ C$ by a perpendicular $v\ L$, meeting $C\ M$ in L. Let $N\ D$ intersect $A\ J$ in α, and let $M\ C$ intersect $A\ J$ in β. In $J\ I$ make $J\ \alpha'$, $J\ \beta'$ respectively equal to $J\ \alpha$, $J\ \beta$. Join $N\ \alpha'$, and prolong $N\ \alpha'$ to F. In $N\ F$ make $N\ O$ equal to $N\ M$; join $O\ \beta'$, and prolong $O\ \beta'$ to G. In $O\ G$ make $O\ P$ equal to $M\ L$. From the point N, with the radius $N\ E$, describe the arc $D\ F$; from the point M, with the radius $M\ D$, describe the arc $C\ D$. Draw $L\ w$ parallel to $A\ I$. From the centre L, with the distance $L\ C$, describe the arc $w\ C$. Join $w\ A$, and prolong $w\ A$ to meet the arc $w\ C$ in x. Join $x\ L$, intersecting $A\ J$ in K. In $J\ I$ make $J\ Q$ equal to $J\ K$; join $P\ Q$, and prolong $P\ Q$ to H. From the centre O, with the radius $O\ F$, describe the arc $F\ G$; from the centre P, with the radius $P\ G$, describe the arc $G\ H$, and from the centre Q, with the radius $Q\ H$, describe the arc $H\ I$. Then the curve $A x C D E F G H I$, comprised by the circular arcs $A\ x$, $x\ C$, $C\ D$, &c., will be a very near approximation to the semi-ellipse of which the axis major is $A\ I$, and the semi-axis minor $J\ E$; since the points A, B, C, &c., are in the curve.

Observations.

The great utility of Problems XIII and XIV is in describing the elevation of a semi-elliptic arch. Having described the curve *A B C D E G* (Fig. 2) divide it into as many equal parts as the arch stones are in number, observing that this number should always be odd, on account of the key-stone. In this example this number of stones are 29. Therefore divide the curve *A E I* into twenty-nine equal parts. Prolong *N E* to *e*. Make *E e* equal to the breadth of the arch stones; prolong *O F* to *f*; *P G* to *g* and *Q H* to *h*. From the centre *N* describe the arc *e f*; from the centre *O* describe the arc *f g*; from the centre *P* describe the arc *g h*, and from the centre *Q* describe the arc *h i*. The same is to be understood of the other side on the right hand of the semi-axis minor *E J*. The centre *N* will serve to draw all the joints of the stones through the points of division in the arc *D E F*; the centre *O* through the points in the arc *F G*; the centre *P* through the points in the arc *G H*, and the centre *Q* through the points in the arc *H I*. The same is to be understood of the right hand side.

In (Fig. 1) the semi-axis major *M A* or *M G* will measure from the scale of equal parts about $33\frac{1}{2}$ say feet, and the semi-axis minor *M D* about 19 from the same scale. In this case the radius of curvature may be found in the following manner, viz. as the semi-axis minor, is to the semi-axis major, so is the semi-axis major to a third proportional which is the radius of curvature *D J*; that is

$M D : M A :: M A : D J$, the radius of curvature;

that is $19 : 33{\cdot}5 :: 33{\cdot}5 : 59$ feet equal to *D J* very nearly.

Therefore having drawn *D J* (as in Prob. XII); make *D J* equal to 59 feet from the same scale.

The distance *A H* or *G L* is a third proportional to the semi-axis major and the semi-axis minor; hence

$M A : M D :: M D : A H$ or $G L$ the radius of curvature;

that is $33{\cdot}5 : 19 :: 19 : 10{\cdot}7 = A H$ or $G L$ very nearly.

Therefore make *A H* or *G L* each equal to 10·7 feet, and proceed in all other respects as directed in Problem XII.

SECTION II.

ON THE CYLINDER, ITS SECTIONS, AND DEVELOPMENT.

Definitions.

If a rectangle revolve about one of its sides until the opposite side comes into its first position, the solid comprised is called *a cylinder.*

The fixed line is called *the axis.*

The surface described by the side of the rectangle which is parallel to the axis is called the *cylindric surface,* or some times *the curved surface.*

The circles described by the two sides of the rectangle which are perpendicular to the axis are called *the bases or ends of the cylinder.*

A portion of a cylinder cut off by a plane parallel to the axis is called *the segment of a cylinder ;* if the plane pass through to the axis the segment is a semi-cylinder.

In this treatise, the segment of a cylinder never exceeds the semicylinder; but is generally less than the half.

The plane of the segment of a cylinder, which is parallel to the axis, is called the *springing plane.*

The straight lines in which the springing plane intersects the curved surface of the cylinder are called the *springing lines.*

The section of the segment of a cylinder, perpendicular to the axis or perpendicular to the springing lines, is called the *right section.*

CONVENTIONAL PRECEPTS AND LEADING PRINCIPLES.

The springing lines are parallel to each other.

A straight line in the chord plane parallel to one of the springing lines is parallel to the axis of the cylinder, and a straight line which is parallel to the axis of the cylinder is parallel to the springing lines.

The centering of any arch treated of in this treatise must be understood to be made in form of the segment of a cylinder, the intrados of the arch being the reverse of the curved surface. The springing plane of the segment is always parallel to the plane of the horizon. The surfaces of the tops or under edges of the beams to which the ribs are fixed are placed in the springing plane.

The section of a cylinder cut by a plane perpendicular to the axis is a circle.

The section of the segment of a cylinder, cut by a plane perpendicular to the axis, is the segment of a circle, and the section of the springing plane is the chord of the arc.

All parallel sections of the segment of a cylinder, cut by a plane either parallel or obliquely to the axis, are equal to one another, and if one was laid upon the other they would coincide.

If a cylinder be cut by a plane obliquely to the axis, the section is an ellipse of which the longest diameter is the axis major and the shortest the axis minor and the axis minor of the ellipse is perpendicular to the axis of the cylinder, and since the two axes of an ellipse are at right angles to each other, if the axis of the cylinder be horizontal and the axis minor of the ellipse perpendicular to the horizon, the axis major of the ellipse shall be parallel to the horizon, and the acute angle made by the axis of the cylinder, and the semi-axis major of the ellipse is equal to the angle of inclination which the axis of the cylinder has to the plane of section; therefore if the cylinder be cut by another plane passing through its axis and through the axis major of the ellipse, the section of the solid will be a springing plane passing through the axis of a cylinder, and the acute angle made by one of the springing lines, and the axis major of the ellipse shall be equal to the inclination of the axis of the cylinder to the oblique plane of section.

Hence if a segment of a cylinder be cut by a plane obliquely to the axis and at the same time perpendicular to the springing plane, the section will still be that of a portion of an ellipse cut off by a double ordinate parallel to the axis-major, and the abscissa apart of the semi-axis minor equal to the height of the right section of the segment of the cylinder.

If a straight line in the chord plane be drawn parallel to the springing lines or the axis of the cylinder, and if two points be taken in this line, two straight lines drawn from each of the points perpendicular to the chord plane to meet the cylindric surface are equal to each other.

If the segment of a cylinder be cut by two planes one perpendicular and the other oblique to the axis, and if a straight line be drawn on the chord plane parallel to the axis to meet the sectional lines, and if from the point of meeting a straight line be drawn in the right section perpendicular to the chord, and in the oblique section perpendicular to the double ordinate, to meet the curved surface, these perpendiculars shall be equal to one another.

In this treatise it must be understood that when the segment of a cylinder is cut by a plane obliquely to the axis, the plane of section is always perpendicular to the springing plane.

PROB. XIV.—Given the right section of a cylindric arch, with radiating joints, the section of the cylindric surface being the arc of a circle not greater than a semicircle, and the angle of obliquity to find the oblique section.

Let $A\ B\ C$, Fig. 1 and Fig. 2, be the right section, draw $C\ L$ and $A\ K$ each perpendicular to $A\ C$. In $C\ L$ take any convenient point L and make the angle $C\ L\ K$ equal to the angle of obliquity.—Divide the arc $A\ B\ C$ into any number of equal parts at 1, 2, 3, &c., and from the points 1, 2, 3, &c., draw straight lines to radiate to the centre I, and these lines will represent the joints. Again from the points 1, 2, 3. &c., draw 1 b, 2 c, 3 d, &c., intersecting $A\ C$ perpendicularly in the points α, β, γ, &c., meeting $K\ L$ in b, c, d, &c. Perpendicular to $K\ L$ draw $b\ e$, $c\ f$, $d\ g$, &c., and make $b\ e$, $c\ f$, $d\ g$, &c., respectively equal to α 1, β 2, γ 3, &c.; and from K, through the points e, f, g, draw the curve $K\ e\ f\ g \ldots q \ldots L$, which is the section of the curved surface of the cylinder. In Fig. 1 the arc $A\ B\ C$ being a semi-circle the centre I is in the diameter $A\ C$. In Fig. 2 the arc being less than a semi-circle, the centre I will fall without the segment; therefore draw $I\ P$ to meet the chord $A\ C$ perpendicularly in P, and bisect $K\ L$ in p. Draw $p\ i$ perpendicular to $K\ L$, and make $p\ i$ equal to $P\ I$. From i through the points K, e, f, g, &c., draw the radiating lines $K\ h$, $e\ l$, $f\ m$, &c., which are the joint lines of the oblique section.

DEMONSTRATION.

The circular arc $A\ B\ C$ being the right section of the intrados or concave cylindric surface of the underside of the arch, and $A\ C$ the chord of the arc $A\ B\ C$; draw $A\ K$ and $C\ L$ each perpendicular to $A\ C$, and let $L\ K$ in the chord plane be the line of section. Then because by construction the straight lines $\alpha\ b$, $\beta\ c$, $\gamma\ d$, &c., meeting $A\ C$ in the points α, β, γ, &c., and $K\ L$ in the points b, c, d, &c., are parallel to $C\ L$ or $A\ K$, and because α 1, β 2, γ 3, &c., are perpendicular to $A\ C$, and because $b\ e$, $c\ f$, $d\ g$, &c., are perpendicular to $K\ L$, and because $b\ e$, $c\ f$, $d\ g$, &c., are respectively equal to α 1, β 2, γ 3, &c. Suppose now the arc $A\ B\ C$ to be raised upon its chord, $A\ C$ in a plane perpendicular to the plane $A\ C\ L\ K$, and suppose the curve $K\ e\ f\ g \ldots L$ to be raised upon the sectional line $K\ L$ in a plane likewise perpendicular to the chord plane $A\ C\ K\ L$; it is evident that the points e, f, g, &c., are not only in the plane of section, but are in also in the surface of the cylinder; for planes passing through $\alpha\ b$, $\beta\ c$, $\gamma\ d$, &c., perpendicular to the plane $A\ C\ L\ K$, would cut the segment of the cylinder in rectangles, of which the ends would be the ordinates; viz. $b\ e$ equal to α 1, $c\ f$ equal to β 2, $d\ g$ equal to γ 3, &c.

PROB. XV.—To find the development of the curve of the oblique section of the segment of a cylinder, the angle of obliquity being given.

Let *A B C* (Fig. 1) be a right section of the segment of the cylinder, *A C* being the chord, and *z B* the height of the arc *A B C.* Draw *C H* perpendicular to *A C*, and make the angle *C A H* equal to the complement of the angle of obliquity. Prolong *C A* to *D*, and make *A D* equal to the length of the arc *A B C* (Prob. v., Sec. 1). Draw *D E* perpendicular to *A D*; make *D E* equal to *C H*, and join *A E*. Divide *A E* into any number of equal parts at the points 1, 2, 3, &c., and parallel to *D E* draw 1-1, 2-2, 3-3, &c., meeting *A D* in 1, 2, 3, &c. Divide the arc *A B C* into the same number of equal parts as the straight line *A E*, and from the points 1, 2, 3, &c., in the arc *A B C* draw the lines 1 a, 2 b, 3 c, &c., intersecting *A C* perpendicularly in *a*, *b*, *c*, &c., and meeting *A H* in a, b, c, &c. From the points 1, 2, 3, &c., in *A D*, make 1 α, 2 β, 3 γ, &c., respectively equal to *a* a, *b* b, *c* c, &c., and from the point *A* through the points α, β, γ, &c., draw the curve *A m E* which will be bisected in *m* by the straight line *A E*.

If *H Z* be drawn parallel and *A Y* perpendicular to *A D* meeting *H Z* in Y, and if *Y Z* be made equal to Y *H*, and if *A Z* be joined a straight line drawn through the point *m* in the curve will be a tangent.

GENERAL PRINCIPLE.

From this operation it is evident that in the development of the surface, the semi-ellipse will become an undulated curve of such a nature that a straight line drawn from one extremity of the curve to the other will divide the curve into two equal parts, the one half being on the one side, and the other half on the other side of the straight line, so that two points being taken in the straight line at equal distances from the point of intersection shall be equally distant from the curve.

EXAMPLE 2, FIGURE 2.

Exhibits the development of the curve which is the oblique section of a semi-cylinder, and is described nearly in the same words as in figure 1, the same letters of reference being used.

OBSERVATION.

In the same manner the development of the curved surface of the segment of a cylinder cut by a cylindric surface of which the axis is perpendicular to the springing plane may be found as is evident from figure 3, and from the explanations given of figure 1.

It sometimes happens in oblique bridges that in order to ease the road way, the ends terminate with circular ends: in such cases this development is useful.

SECTION III.

Definitions.

1. If a limber plane surface, or paper, of which the outline is a right-angled triangle, be rolled upon the curved surface of a cylinder so as to have no vacuity, and that one of the sides about the right angle not greater than the circumference of the cylinder, may be in a plane perpendicular to the axis, the curve line assumed by the hypothenuse becoming bent, is called *a cylindric spiral.*

From this definition it is obvious, that of the two sides about the right angle of the triangle thus bent, the one which falls upon the circumference will be an arc of a circle, having the same radius as the cylinder, and the other a straight line parallel to the axis.

2. The axis of the cylinder is also called *the axis of the spiral.*

3. The radius of the cylinder is called *the radius of the spiral.*

4. If the limber surface be unrolled so as to coincide with a plane, the figure again resumes its triangular form, and is called *the triangle of development.*

5. The side of the triangle, which was applied to the circumference of the base of the cylinder, is called *the base of development.*

6. The side of the triangle which was parallel to the axis of the cylinder, is called *the perpendicular of development.*

7. The hypothenuse is called *the development of the spiral line.*

8. If a straight line be supposed to move perpendicular to the axis of a cylinderic spiral, continually touching the spiral, the surface generated is called *a spiral surface,* whether it lies between the axis and the spiral line or on the other side of the spiral line.

If another cylindric surface, about the same axis, be supposed to intersect the spiral surface, the line of intersection of the two surfaces will be another spiral of which the triangle of development shall have its perpendicular equal to the perpendicular of the triangle of development of the first spiral, and the lengths of the bases of the two developments will have the same ratio to one another as the radii of their respective cylindric surfaces.

9. In the triangles of development of the two spirals belonging to the same spiral surface, the difference of the angles made in each by the hypothenuse and perpendicular is called *the angle of the twist.*

General Principles.

If a spiral surface be cut by a plane, either perpendicular to, or passing along the axis, the section will be a straight line perpendicular to the axis.

If a cylinder be cut by a plane obliquely to its axis, one straight line on the cutting plane will be perpendicular to, or make a right angle with, the axis, and this line is the semi-axis minor of the ellipse, which is the section of the curved surface.

If a spiral surface be cut by a plane parallel to the axis of the cylinder, the section will be a curve, excepting in the case in which the straight line drawn in the cutting plane perpendicular to the axis meets the spiral surface.

If a spiral surface be cut by a plane obliquely to the axis of the cylinder, the section will be a curve of contrary flexure, and if the spiral surface be cut by another plane passing along the axis perpendicular to the first plane, the section which is a straight line will intersect the curve of contrary flexure in the point of retrogression.

PROB. XVI.—To find the projections of one or more revolutions of two spiral lines which comprise a spiral surface between them, on a plane parallel to the axis of the cylindric surfaces.

Let $A\,E\,I\,J$, $K\,O\,S\,T$ (Fig. 1), be circles of which their centre is V, and their diameters respectively equal to the diameters of the greater and less cylindric surfaces. Divide the circumference of the greater circle into as many equal parts, $A\,B$, $B\,C$, $C\,D$, &c., as the number of points to be found in the projection of each spiral in one revolution. From the points A, B, C, &c., of division, draw lines tending to the centre V, meeting the circumference of the less circle in K, L, M, &c., which will also divide its circumference into the same number of equal parts as the greater circumference. In the same straight line, with the centre V, draw $W\,X$ to represent the axis. In $W\,X$ make $W\,Y$ equal to the length which the axis will have in half a revolution, and draw $a\,k$ in a straight line with W, perpendicular to $W\,X$. Divide $W\,Y$ at the point 1, 2, 3, &c., into eight equal parts, the number of points to be found in half a revolution; and through the points 1, 2, 3, &c., draw the lines $b\,l$, $c\,m$, $d\,n$, &c., parallel to $a\,k$. Draw $A\,a$, $B\,b$, $C\,c$, &c., parallel to $W\,X$, and from the point a draw a curve through b, c, d, &c., which will be the projection of the exterior cylindric spiral. Draw $K\,k$, $L\,l$, $M\,m$, &c., also parallel to $W\,X$, and from the point k, draw another curve through the the points l, m, n, &c., which will be the projection of the spiral of the interior cylindric surface.

In Fig. 2, draw the straight line $P\,R$, and draw $P\,O$ perpendicular to $P\,R$. Make $P\,O$ equal to the length of four parts along the axis; make $P\,R$ equal to the arc $A\,B\,C\,D\,E$ of a quarter of the circumference of the exterior cylinder, and draw $O\,R$. In $P\,R$ make $P\,Q$ equal to the arc $K\,L\,M\,N\,O$ of a quarter of the circumference of the interior cylindric surface, and draw $Q\,O$; then the angle $Q\,O\,R$ is the angle of the twist.

The curve $e\,a\,g$ (Fig. 3), drawn as shown from the adjacent quadrant $A\,B\,C\,D\,E$ is the same as the curve $u\,i\,h\,g\,f\,o$ (No. 2) found by projection, and the curve $O\,k\,Q$ drawn as shown from the adjacent quadrant $K\,L\,M\,N\,O$ is the same as the inner curve $u\,s\,r\,g\,p\,c$. Each of the curves thus drawn by means of a quadrantal arc is called the figure of the sines or a sinical curve, therefore the projection of a spiral line is either a sinical curve or formed by two or more sinical curves.

OBSERVATIONS.

The construction of the spiral surface, now explained, will be truly exemplified in a spiral stair; supposing the plan to consist of 16 steps in the whole circumference. The length of the axis corresponding to this number will be 16 times the height of one step. A curve line drawn upon the cylindric surface of the wall through the points of intersection of the lines in which the sides forming the interior angles of the steps meet each other, will form the spiral line upon the concave cylindric surface of the wall of the stair, and if another concentric cylindric surface of which the radius is equal to that of the inner circle of the plan be supposed to exist, a curve line drawn upon the convex surface through the points of intersection of the lines in which every two sides forming the interior angles of the steps meet each other will form the other spiral line next to the well-hole.

In this manner the spiral pump, the invention of which is attributed to Archimedes, may be constructed.

PROB. XVII.—Given the angle of revolution of a spiral surface, the radius, and the length of each spiral, to find the projection of the spiral surface, and to find the angle of the twist.

Draw the straight line $V A$ (Fig. 1). Make $V A$ equal to the radius of the outer spiral, and $V K$ equal to that of the inner spiral. From the centre V, with the radius $V A$, describe the circle $A E I$, and make the angle $A V I$ equal to the angle of revolution. From the centre V, with the distance $V K$, describe the arc $K O S$, meeting $V I$ in S. Divide the arc $A E I$ into any convenient number of equal parts, as here into eight. In this example the number of parts are even. Through the middle E draw $V E$, and draw $P Q$ perpendicular to $V E$. Let the points of division in the arc $A E I$ be B, C, D, &c. Draw the straight lines $B L$, $C M$, $D N$, &c., radiating to the centre V, meeting the inner arc $K O S$ in the points L, M, N, &c., Prolong $V E$ to Z, and make $E Z$ equal to the length of the spiral. Divide $E Z$ into as many equal parts as the arc $A E I$, viz., eight, and through the points 1, 2, 3, &c., draw the straight lines $l b$, $m c$, $n d$, &c., parallel to $P Q$. Draw $A a$, $B b$, $C c$, &c, perpendicular to $P Q$, so that $A a$ may meet $P Q$ in a. Also perpendicular to $P Q$ draw $K k$, $L l$, $M m$, &c., so that $K k$ may meet $P Q$ in k. From the point a, through the points b, c, d, &c., draw the curve line $a b c d e f g h i$, and from the point k, through the points l, m, n, &c., draw the curve line $k l m e p q r s$, and these curves are the projections of the two spirals which comprise the spiral surface.

In figure 2 draw $U X$, and $U Y$ perpendicular to $U X$. Make $U Y$ equal to four parts of $E Z$. Make $U X$ equal to the length of the arc $A E$, and in $U X$ make $U W$ equal to the length of the arc $K O$. Draw $Y X$ and $Y W$; then $W Y X$ is the angle of the twist.

If a straight line were a tangent to the curve $a e i$, at the point e (fig. 1) and another straight line a tangent to the curve $k e s$ at the same point e; the opposite angles made by these two straight lines would be equal to the angle of the twist.

PROB. XVIII.—To find the sections of a spiral surface cut by two planes which are parallel to each other, and parallel to the axis of the cylinder, and at a given distance from the axis, and to find the angle of the twist made by the curves, which are the sections of the spiral surface.

Let the arcs k o r and *K E T* be each respectively a section of the interior and exterior cylindric surface by a plane perpendicular to the axis. Draw the radiating lines *T* r, *K* k, towards the centre s. Let the parallel lines *A U* and *K* T be the sectional lines, *A U* being a tangent at *E*, the middle of the arc *K T*, and *K T* a tangent at *o*, the middle of the arc k r. Divide the arc *K E T* into any number of equal parts at the points 1, 2, 3, &c., as into eight. Prolong k *K* to *A*, and r *T* to *U*, and through the points, 1, 2, 3, &c., in the arc *K E T*, draw radiating lines, meeting the arc k o r in the points l, m, n, &c., and the straight line *A U* in the points *B*, *C*, *D*, &c., also intersecting the straight line *K T* in *L*, *M*, *N*, &c. In *E e* make *E* α, α β, β γ, &c., equal to each other, and each equal to the length of the axis which the spiral advances upon the surface from one meridian to the next, the meridians being straight lines on the surface of the cylinder parallel to the axis at distances from each other equal to the arcs *K* 1, 1–2, 2–3, &c., of the arc *K E T*. Parallel to *A U* draw α *b*, β *c*, γ *d*, &c., and parallel to *E e* draw *B b*, *C c*, *D d*, &c.—From *A*, through the points *b*, *c d*, &c., draw the curve *A b c...e...i*. Parallel to *E e* draw *L l*, *M m*, *N n*, &c., meeting α *b*, β *c*, γ *d*, &c.. in the points *l*, *m*, *n*, &c.; also parallel to *E e* draw *K k*, meeting *A U* in *k*. and from *k*, through the points *l*, *m*, *n*, &c., draw the curve *k l m...e...s*; then the curves *A b c...e...i*, and *k l m...e...s*, are the sections of the spiral surface.

In Fig. 2, draw the straight line *V X*. Make *V X* equal to the length of the four arcs *K* 1, 1–2, 2–3, &c., that is equal to the length of the arc *K E* or *E T*. and draw *V Y* perpendicular to *V X*. Make *V Y* equal to *E e* (Fig. 1), and join *X Y*. In *V X* make *V W* equal to the four arcs k l, l m, m n, &c.; that is equal to the length of the arc *k o*, or *o r*, and join *Y W*; then the angle *W Y X* is the angle of the twist.

If a straight lines be drawn through the point *e* (Fig. 1) respectively parallel to *Y X*, *Y W* (Fig. 2), they shall be tangents to the two curves at the point *e*. Therefore each of the opposite angles made by the two tangents, shall be equal to the angle of the twist.

OBSERVATION.

A stair is said to be right handed when the steps begin to ascend upon the right hand of the newel or well-hole. With regard to spirals, a common cork-screw is a right-hand spiral. In a right-hand stair, the spirals formed upon the cylindric surface of the wall, and upon the cylindric surface of the well-hole, are right-hand spirals.

SECTION IV.

ON THE TRIHEDRAL.

DEFINITIONS.

Def. 1. The solid angle made by three plane angles is called *a trihedral.* Thus the three faces of a triangular pyramid is a trihedral.

Def. 2. The angle made by two plane faces of a solid is called *a Dihedral angle.* The measure of a dihedral angle is the angle contained by the straight lines, one drawn upon each face perpendicular to the line in which the two faces meet from the same point.

Def. 3. If two of the faces of a trihedral be perpendicular to each other, the trihedral is called *a right trihedral.*

Def. 4. The angle made by the edges of any face is called *the angle of that face.*

Def. 5. The two faces which are perpendicular to each other, are called *the right faces*, and the remaining face is called *the oblique face.*

Def. 6. The edge between the two right faces is called *the right edge*, and the other two edges are called *the oblique edges.*

If a trihedral be cut by a plane perpendicular to one of its oblique edges, the section shall be a right angled triangle, and each of the three sectional lines shall be perpendicular to one of the two lines which contain the angle of the face cut by that sectional line, viz. two of the sectional lines shall be perpendicular to the edge to which the cutting plane was perpendicular, and the remaining sectional line upon the opposite face perpendicular to the right edge.

Hence the three triangles made by the sectional triangle, and the sectional triangle itself shall be all right angled triangles.

Def. 7. The oblique edge to which the cutting plane is perpendicular, is called *the adjacent edge.*

Def. 8. The right face which has the adjacent edge for one of the lines containing the angle of that face is called the adjacent face.

Def. 9. The face opposite the adjacent edge is called *the opposite face.*

The trihedral consists of five parts, viz., the angles of the three faces and the two acute dihedral angles. Any two of the five parts being given, the three remaining parts may be found; the cases being similar to those of a right angled triangle.

GENERAL PRINCIPLES.

In every case it is necessary to make one of the right faces the adjacent face. When a dihedral angle is a given, or required part, the right face adjacent to the dihedral angle must be the adjacent face, and the other opposite to this angle the opposite face.

The sectional line upon the adjacent face must always be perpendicular to the adjacent edge, and the sectional line upon the opposite face perpendicuar to the right edge; moreover, the sectional line

upon the oblique face, must, as well as the sectional line upon the right face, be perpendicular to the adjacent edge.

If any two parts of a right trihedral be given, the like parts of the sectional triangle may be found, and if the like parts of the sectional triangle be given, the remaining parts of the trihedral may be found.

When the angles of two faces of a trihedral are joined together so as to make one angle equal to their sum, these angles must either be the angles of the two right faces or the angles of the oblique and adjacent faces, or the angles of the oblique and opposite faces. In any of the three cases, the radius must always be made upon the connecting line.

It would occupy too much space to explain the properties of all the cases of the trihedral, which are six in number; we shall, therefore, only give the propositions in which the two right faces are concerned being absolutely necessary.

Proposition 1.

The angle of the oblique face is equal to the acute angle of a right-angled triangle, contained by the two lines of which one is equal to the cosine of the angle of the adjacent face, and the other equal to the secant of the angle of the opposite face.

Proposition 2.

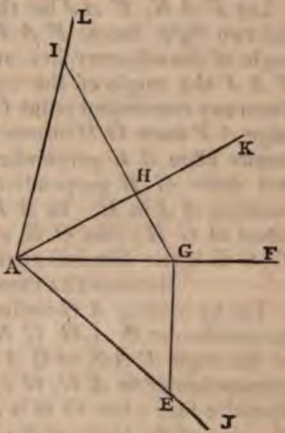
The two sides of the sectional triangle, which contain the right angle, shall be respectively equal to the sine of the angle of the adjacent face, and the tangent of the angle of the opposite face, and the dihedral angle of the trihedral shall be equal to the angle of the sectional triangle contained by the hypothenuse, and the side equal to the sine of the angle of the adjacent face.

Remark.

An angle is generally known to the mason by the name of a bevel, and is transferred from one place to another by means of an instrument of the same name, which is so well understood by workmen as not to require description. Bevels are, however, of two kinds, one of which is drawn upon a surface, and the other is the angle made by two surfaces, which we have here called a dihedral angle. It must be observed, that in taking this angle, when the two legs of the level are sufficiently extended, the inner angle, in which the inner edges meet, must rest upon the arris of the stone with one of the inner edges upon the one surface, and the other upon the other surface, while each edge of the bevel is perpendicular to the arris of the stone. The same is to be observed in working one surface, the other being already wrought.

A dihedral angle may be either greater or less than a right angle, but if the acute angle be given, the obtuse angle will be found by deduction or subtraction as it is the complement of two right angles.

PROB. XIX.—Given the angles of the two right faces of the trihedral to find the angle of the oblique face.



Let $F A K$, $F A J$, be the angles of the two right faces, $F A K$ being the angle of the adjacent face, and therefore $F A J$ the angle of the opposite face.

From any convenient point G, in the right edge, $A F$, draw $G I$ intersecting the adjacent edge $A K$ perpendicularly in H, and draw $G E$ perpendicular to $A F$, meeting $A J$ in E. From the point A, with the distance $A E$, cut $H I$ in I, and through I draw $A L$; then the angle $K A L$, or $H A I$, is the angle of the oblique face.

For by making $A G$ radius, $G H$ being perpendicular to $A H$, $A H$ is the cosine of the angle $G A H$ of the adjacent face, and $G E$ being perpendicular to $A G$, $A E$ is the secant of the angle $G A E$ of the opposite face; but $A I$ is equal to $A E$; therefore the angle $H A I$ of the right-angled triangle $A H I$ being contained by the sides $A H$, $A I$ respectively equal to the cosine of the angle of the adjacent face and the secant of the angle of the opposite face, is equal to the angle of the oblique face.

ILLUSTRATION.

Suppose the plane of the triangle $A G E$ to be raised perpendicular to the plane of the triangle $F A K$, and the triangle $H A I$ to be turned upon $A H$ as a hinge, until the point I fall upon E, and the straight line $A I$, or $A L$, upon $A E$, or $A J$, and the three plane angles thus united will form the trihedral solid angle, and the ends of the three faces will be a right-angled triangle, and is what is here called the sectional triangle.—*See* the demonstration, Prob. XXI.

REMARK.

Let $F A K$ be the angle which the upper edge of the hip rafter of a roof makes with its base in a vertical plane, $A K$ being the hip line, and $A F$ the base or plan, and let $F A J$ be the angle which the base of the hip line makes with the edge of the wall-plate in a horizontal plane; then $K A L$ is the angle which the hip line makes with the wall-plate.

The method of finding the angle of any face having the angles of the other two faces given; or of finding a third part, two parts being given, will be found in my Book of Dialling, pages 3, 4, 5, 6, in which the equations are investigated.

Prob. XX.—Given the angles of the two right faces to find the dihedral angle adjacent to one of them.

Let $F\,A\,K$, $F\,A\,J$ be the angles of the two right faces, $F\,A\,K$ being the angle of the adjacent face, and therefore $F\,A\,J$ the angle of the opposite face. From any convenient point G in the right edge $A\,F$ draw $G\,H$ intersecting the adjacent edge $A\,K$ perpendicularly in H, and draw $G\,E$ perpendicular to $A\,F$ meeting $A\,J$ in E. In $A\,F$ make $G\,O$ equal to $G\,H$; join $O\,E$, and the angle $G\,O\,E$ is the dihedral angle required.

Demonstration.

For by making $A\,G$ radius $G\,H$ being perpendicular to $A\,H$, $G\,H$ is the sine of the angle $F\,A\,K$ or $G\,A\,H$ of the adjacent face, and $G\,E$ being perpendicular to $A\,G$, $G\,E$ is the tangent of the angle $G\,A\,E$ of the opposite face; but $G\,O$ is equal to $G\,H$; therefore $G\,O$ is equal to the sine of the angle of the adjacent face; hence the two sides $G\,O$, $G\,E$ which contain the right angle of the sectional triangle $G\,O\,E$ are respectively equal to the sine of the angle of the adjacent face and the tangent of the angle of the opposite face, and the dihedral angle equal to the angle $G\,O\,E$ contained by the hypothenuse and the side $O\,G$ equal to the sine of the angle of the adjacent face.

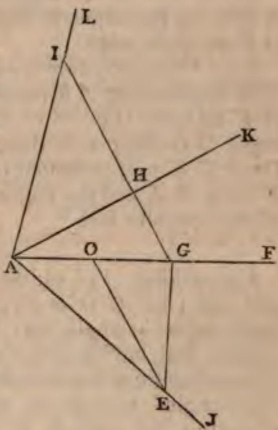
Or thus.

For let the plane containing the angle $G\,A\,H$, or $F\,A\,K$, be raised upon the straight line $A\,F$, as a hinge, until it becomes perpendicular to the plane containing the angle $F\,A\,J$, or $G\,A\,E$, and let the plane containing the triangle $O\,G\,E$ be made to revolve upon the line $G\,E$ until the side $G\,O$ fall upon $G\,H$; and because $G\,O$ is equal to $G\,H$, the point O will coincide with the point H; therefore the straight line $O\,E$ will coincide with a straight line drawn from H to E; and because the point H is on one edge of the oblique face, and the point E on the other edge, the straight line $O\,E$ will be on the oblique face perpendicular to the adjacent edge $A\,K$, and because $O\,E$ and $H\,G$ are drawn, one in the plane of the oblique face, and the other in the plane of the right face, each perpendicular to $A\,K$, the line of common section of these two planes, the angle $G\,O\,E$ is the dihedral angle required.

Remark.

Let $F\,A\,K$ be the angle which the upper edge of the hip rafter of the roof makes with its base in a vertical plane, $A\,K$ being the hip line and $A\,F$ its base or plan, and let $F\,A\,J$ be the angle which the base of the hip line makes with the edge of the wall-plate in a horizontal plane; then $G\,O\,E$ or $F\,O\,E$ is the dihedral angle of the back.

Prob. XXI.—Given the angles of the two right faces to find the angle of the oblique face, and the dihedral angle adjacent to either of the right faces in one operation.



Let $F\,A\,K$ be the angle of the adjacent face, and therefore $F\,A\,J$ the angle of the opposite face.

From any convenient point G in the right edge $A\,F$, draw $G\,I$, intersecting the adjacent edge $A\,K$ perpendicularly in H, and draw $G\,E$ perpendicular to $A\,F$, meeting $A\,J$ in E. From the point A with the distance $A\,E$, cut $H\,I$ in I, through I draw $A\,L$; then the angle $K\,A\,L$ or $H\,A\,I$ is the angle of the oblique face.

In $A\,F$ make $G\,O$ equal $G\,H$; join $O\,E$ and the angle $G\,O\,E$ is the dihedral angle required.

Demonstration.

For $A\,I$ is equal to $A\,E$, and $G\,O$ equal to $G\,H$;

Let $A\,I = A\,E = a$, $G\,E = b$, and $G\,O = G\,H = c$.

Then by Euclid, Book 1, Proposition 47:—

$$A\,G^2 = A\,E^2 - G\,E^2 = a^2 - b^2$$
$$A\,H^2 = A\,G^2 - G\,H^2 = a^2 - b^2 - c^2$$
$$H\,I^2 = A\,I^2 - A\,H^2 = a^2 - (a^2 - b^2 - c^2) = b^2 + c^2$$
$$O\,E^2 = E\,G^2 + O\,G^2 = b^2 + c^2$$

Therefore $H\,I$ is equal to $O\,E$.

Suppose the plane containing the angle $G\,A\,H$, or $F\,A\,K$, to be revolved upon the straight line $A\,F$ until it becomes perpendicular to the plane containing the angle $F\,A\,J$, or $G\,A\,E$, and let the plane containing the angle $O\,G\,E$ be revolved upon the straight line $G\,E$ until the edge $G\,O$ fall upon $G\,H$; and because $G\,O$ is equal to $G\,H$, the point O will coincide with the point H, and the straight line $O\,E$ will, by hypothesis, be perpendicular to $A\,K$ at the point H; moreover, let the plane containing the angle $K\,A\,L$, or $H\,A\,I$, revolve upon $K\,A$ until the edge $A\,I$, or $A\,L$, fall upon the plane containing the angle $F\,A\,J$, or $G\,A\,E$, and because $H\,I$ is perpendicular to $A\,K$, and $O\,E$ is, by hypothesis, also perpendicular to $A\,K$ from the point H, the straight line $H\,I$ will fall upon $O\,E$; and because $H\,I$ is equal to $O\,E$, the point I will fall upon E, and the straight line $A\,I$ upon $A\,E$.

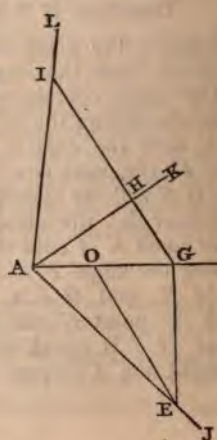
Prob. XXII.—Given a dihedral angle of the trihedral and the angle of the adjacent face to find the angle of the opposite face.

Let $F\ A\ K$ be the angle of the adjacent face, $A\ K$ being the adjacent edge, and consequently $A\ F$ the right edge.

In $A\ F$ take any convenient point G, and draw $G\ H$ intersecting $A\ K$ perpendicularly in H. In $A\ F$ make $G\ O$ equal to $G\ H$ and make the angle $G\ O\ E$ equal to the given dihedral angle. Draw $G\ E$ perpendicular to $A\ G$, and through E draw $A\ J$, and the angle $F\ A\ J$ to the angle of the opposite face.

For making $A\ G$ radius, $G\ H$ is the sine of the angle of the adjajent face; but $G\ O$ is equal to $G\ H$, and the angle $G\ O\ E$ of the right-angled triangle $O\ G\ E$ is equal to the dihedral angle of the trihedral, and having the side $G\ O$ and an angle equal to the given dihedral angle of the trihedral, viz., a side and one of the acute angles, a right-angled triangle is easily constructed, as already done in constructing the triangle $O\ G\ E$; but $A\ G$ being radius, $G\ E$ is the tangent of the angle $G\ A\ E$; therefore the two sides $O\ G, G\ E$, which contain the right angle $O\ G\ E$, are respectively equal to $G\ H$, the sine of the angle $G\ A\ H$ of the adjacent face, and to $G\ E$ the tangent of the angle of the opposite face; therefore the right-angled triangle $O\ G\ E$ is the sectional triangle.

Prob. XXIII.—Given a dihedral angle of the trihedral, and the angle of the adjacent face, to find the angle of the opposite face and the angle of the oblique face in one operation.



Let $F\ A\ K$ be the angle of the adjacent face, $A\ K$ being the adjacent edge, and consequently $A\ F$ the right edge.

In $A\ F$ take any convenient point G, and draw $G\ I$ intersecting $A\ K$ perpendicularly in H. In $A\ F$ make $G\ O$ equal to $G\ H$, and make the angle $G\ O\ E$ equal to the given dihedral angle. Draw $G\ E$ perpendicular to $A\ G$, and through E draw $A\ J$, and the angle $F\ A\ J$ is the angle of the opposite face.

From A, with the distance $A\ E$, cut $H\ I$ in I, and through I draw $A\ L$, and the angle $K\ A\ L$ is the angle of the oblique face.

SECTION V.

Preliminary Calculations.

TABLE OF THE LENGTHS OF CIRCULAR ARCS.

In the execution of oblique arches it is necessary to find the development of the intrados; but before this development can be made it is necessary to find the length of the circular arc, which is a section of the cylindric centre perpendicular to the axis. There are no rules by which the length of a circular arc can be found with sufficient exactness. The following table which I computed early in the year 1827, contains a series of circular arcs in all proportions; that is to say, if an arc of a circle is required to be executed which shall have a given chord or span, and a given height not exceeding half the chord, an arc will be found in tables which shall have the same proportion; then, because the corresponding lines of similar figures are proportional, it will be, as the chord of the tabular arc is to the chord of the required arc, so is the length of the curve of the tabular arc to the length of the curve of the required arc; but as the tabular arcs have their chords equal to unity, it will be, as the chord of the given arc is to its height, so is unity the length of the chord of the tabular arc, to the height of the tabular arc; then as unity the chord of the tabular arc, is to the length of the tabular arc so is the given chord to the length of the corresponding arc, and thus we have only to multiply the length of the tabular arc by the given chord, and the product is the length of the arc required. By attending to the following Problem the greatest exactness will be attained in finding the length of the curve.

Height of arc.	Length of arc.	Height of arc.	Length of arc.	Height of arc.	Length of arc.	Height of arc.	Length of arc.	Height of arc.	Length of aic.
·080	1·01698	·104	1·02860	·128	1·04313	·152	1·06051	·176	1·08066
·081	1·01741	·105	1·02915	·129	1·04380	·153	1·06130	·177	1·08156
·082	1·01784	·106	1·02970	·130	1·04447	·154	1·06209	·178	1·08246
·083	1·01827	·107	1·03026	·131	1·04515	·155	1·06288	·179	1·08337
·084	1·01871	·108	1·03082	·132	1·04583	·156	1·06368	·180	1·08428
·085	1·01916	·109	1·03139	·133	1·04652	·157	1·06449	·181	1·08519
·086	1·01961	·110	1·03196	·134	1·04721	·158	1·06530	·182	1·08611
·087	1·02006	·111	1·03254	·135	1·04791	·159	1·06611	·183	1·08704
·088	1·02051	·112	1·03312	·136	1·04861	·160	1·06693	·184	1·08797
·089	1·02099	·113	1·03371	·137	1·04932	·161	1·06775	·185	1·08890
·090	1·02146	·114	1·03430	·138	1·05003	·162	1·06858	·186	1·08984
·091	1·02194	·115	1·03490	·139	1·05075	·163	1·06941	·187	1·09079
·092	1·02242	·116	1·03550	·140	1·05147	·164	1·07025	·188	1·09174
·093	1·02291	·117	1·03611	·141	1·05220	·165	1·07109	·189	1·09269
·094	1·02340	·118	1·03672	·142	1·05293	·166	1·07194	·190	1·09365
·095	1·02389	·119	1·03734	·143	1·05367	·167	1·07279	·191	1·09461
·096	1·02440	·120	1·03797	·144	1·05441	·168	1·07365	·192	1·09557
·097	1·02490	·121	1·03860	·145	1·05516	·169	1·07451	·193	1·09654
·098	1·02542	·122	1·03923	·146	1·05591	·170	1·07537	·194	1·09752
·099	1·02594	·123	1·03987	·147	1·05667	·171	1·07624	·195	1·09850
·100	1·02645	·124	1·04051	·148	1·05743	·172	1·07711	·196	1·09949
·101	1·02698	·125	1·04116	·149	1·05819	·173	1·07799	·197	1·10048
·102	1·02752	·126	1·04181	·150	1·05896	·174	1·07888	·198	1·10147
·103	1·02806	·127	1·04247	·151	1·05973	·175	1·07977	·199	1·10247

PROB. XXIV.—To find the length of the arc of a circle, the chord and height of the arc being given, but the height not to exceed half the chord.

Divide the height of the arc by the chord to three places of decimals: look in the table under "height of arc" for the number which is equal to the quotient: from the next column under "length of arc" take out the opposite number, which will be the length of a similar arc to that which is to be found, having unity for its chord: multiply the tabular length of the arc by the given chord, and the product is the length of the arc to that chord nearly.

But if after having divided the given height of the required arc by its chord, the quotient does not terminate in the third place of decimals, continue, if necessary, to find three more places of decimals; look in the table under "height of arc" for anumber equal to the three first quotient figures; in the column on the right take out the opposite numbers corresponding to the first three figures and to the next greater; multiply the difference by the three remaining figures considered as a decimal; to the first three add the product to the length of the arc corresponding to the three first figures; multiply the sum by the given chord, and the product is the length of the arc required.

EXAMPLE 1.

Required the length of an arc of which the chord is 22 feet and the height 5 feet 6 inches?

22f.×12=264 inches, and (5ft. 6in.)×12=66 inches

66÷264=·250 exactly.

The tabular length of the arc answering to the height ·250 is 1·15912.
and 22×1·15912=25·50064 the length of the arc.

EXAMPLE 2.

Required the length of an arc of which the chord is 38ft. 9in. and the height 6 feet.

Here 38ft. 9in.=465 inches, and 6ft.=72 inches.
and 72÷465=·154|838.

The tabular length of the arc answering to 155 is 1·06288
and the tabular length of the arc answering to 154 is 1·06209
Their difference is ·00079

·00079×·838=·00066 retaining only five places of decimals,
and 1·06209+·00066=1·06275,
and 465×1·06275÷12=41·18156 feet, the answer.

Example 3.

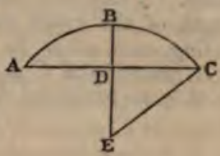
Required the length of an arc of which the chord is 22f. 10′ and the height 8 feet?

22f. 10′=274 inches, and 8f.=96 inches,
and 96÷274=·350|364.
The tabular length of the arc answering to the height 351 is 1·30156
and the tabular length of the arc answering to the height 350 is 1·29997
Their difference is ·00159
·00159×·364=·00058 retaining five places of decimals,
and 1·29997+·00058=1·30055,
and lastly 274×1·30055÷12=29·69589 feet.

Examples for Practice.

1. Required the length of an arc of which the chord is 23 feet and the height 6 feet? *Ans.* 26·96934 *feet.*
2. Required the length of an arc of which the chord is 48 feet 9 inches and the height 14 feet 6 inches? *Ans.* 59·53496 *feet.*
3. Required the length of an arc of which the chord is 17·374 feet and the height 6 feet? *Ans.* 22·45815 *feet.*
4. Required the length of an arc of which the chord is 15·2708 and the height $4\frac{1}{6}$ feet? *Ans.* 18·14155 *feet.*
5. Required the length of an arc of which the chord is 19 feet and the height 6 feet? *Ans.* 23·68426 *feet.*
6. Required the length of an arc of which the chord is 45·96 feet and the height 14 feet 6 inches? *Ans.* 57·3204 *feet.*
7. Required the length of an arc of which the chord is 19 feet and the height 7 feet 1 inch? *Ans.* 25·40262 *feet.*
8. Required the length of an arc of which the chord is 19 feet and the height of the arc 6·8776. *Ans.* 25·06461 *feet.*

PROB. XXV.—Given the chord and height of an arc to find the radius of the circle.



Let $A\ B\ C$ be the arc, $A\ C$ the chord, and E the centre. Draw the radius $E\ B$, bisecting the chord $A\ C$ perpendicularly in D, and join $C\ E$.

Let $h = D\ C$ or $D\ A$ = half the chord, and $a = D\ B$ the height of the arc; and let $x = E\ C = E\ B$, the radius of the circle; then will $D\ E = x - a$.

Then by Euclid, Book I., Proposition 47:—

$$E\ C^2 = E\ D^2 + C\ D^2$$

That is $x^2 = (x-a)^2 + h^2$

Or $x^2 = x^2 - 2a\,x + a^2 + h^2$

Whence $x = \dfrac{a^2+h^2}{2a} = \frac{1}{2}\left(\dfrac{h^2}{a} + a\right)$

RULE IN WORDS.

Divide the square of the half chord by the height of the arc; add the height of the arc to the quotient, and half the sum is the radius of the circle.

EXAMPLE.

Given the chord 22 feet, and the height of the arc 5 feet 6 inches, to find the radius of the circle.

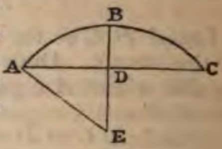
Here $22 \div 2 = 11$ feet, the half chord, and 5ft. 6in. = 5·5feet.

$11^2 = 121$, which, divided by 5·5 feet gives 22 for the quotient, $22 + 5{\cdot}5 = 27{\cdot}5$ and $27{\cdot}5 \div 2 = 13{\cdot}75 = 13$ feet 9 inches, the radius.

EXAMPLES FOR PRACTICE.

1. Required the radius of an arc of which the chord is 38 feet 9 inches and the height 6 feet? *Ans.* 34·2825 *feet.*
2. Required the radius of an arc of which the chord is 22 feet 10 inches and the height 8 feet? *Ans.* 12·1462 *feet.*
3. Required the radius of an arc of which the chord is 23 feet and the height 6 feet? *Ans.* 14·02083 *feet.*
4. Required the radius of an arc of which the chord is 48 feet 9 inches and the height 14 feet 6 inches? *Ans.* 27·7376 *feet.*
5. Required the radius of an arc of which the chord is 17·374 *feet.* and the height 6 feet? *Ans.* 9·2886 *feet.*
6. Required the radius of an arc of which the chord is 15·2708 feet and the height $4\frac{1}{6}$ feet? *Ans.* 9 0792 *feet.*
7. Required the radius of an arc of which the chord is 19 feet and the height 6 feet? *Ans.* 10·52008 *feet.*
8. Required the radius of an arc of which the chord is 45·96 feet inches and the height 14 feet 6 inches? *Ans.* 25·4596 *feet.*

PROB. XXVI.—Given the radius of a circle, to find the height of an arc of that circle, the chord of the arc being given.



Draw the straight line $A\ C$ equal to the given chord. From the points A and C with the length of the given radius describe arcs intersecting each other in E, and draw the radius $E\ B$ bisecting $A\ C$ perpendicularly in D, then $D\ B$ is the height of the arc.

BY CALCULATION.

Join $E\ A$ and let $E\ A = E\ B = r$, $A\ D = C\ D = h$, and $D\ B = x$; then will $D\ E = r - x$.

By Euclid, Book 1., Proposition XLVII.

$$A\ D^2 + D\ E^2 = E\ A^2,$$

Or $h^2 + (r-x)^2 = r^2$,

Or $(r-x)^2 = r^2 - h^2$,

Hence, $r - x = \sqrt{(r^2 - h^2)}$,

By changing the signs,

$$x - r = -\sqrt{(r^2 - h^2)},$$

therefore, $x = r - \sqrt{(r^2 - h^2)}$.

ARITHMETICAL RULE.

From the radius subtract the square root of the difference of the square of the radius and that of the half chord, and the remainder is the height of the arc.

EXAMPLE. 1.

Given, the radius of a circle equal to 36 feet, and the chord of an arc of that circle equal to 4 feet, to find the height of the arc.

Here, $36^2 = 1296$ the square of the radius,
And $2^2 = 4$ the square of the half chord,
1292 the difference of those squares.
$\sqrt{(1292)} = 35·9444$ the square root of the difference,
$36 - 35·9444 = 0·0556$ the height of the arc.

EXAMPLE 2.

Given the radius of a circle equal to 726 feet, and the chord of an arc of that circle equal to 4 feet, to find the height of the arc.

Here $726^2 = 527076$ the square of the radius,
And $2^2 = 4$ the square of the half chord,
527072 the difference of their squares,
$\sqrt{(527072)} = 725·9972$ the square root of the difference.
$720 - 725·9972 = 0·0028$ the height of the arc.

EXAMPLE FOR PRACTICE.

Required the height of an arc of a circle of which the radius is 10 feet, to a chord of 19 feet? *Ans.* 6·87751.

Let the parallelogram *A G Q H* be the plan of an oblique arch. Two of the opposite sides *A H, G Q* are the lengths of the faces upon which the arch-way terminates, and the other two *A G, H Q* are the lengths of the springers or abutments. The acute angle *A H Q* or *A G Q* is that which is called the angle of obliquity. If a straight line be drawn from the vertex *A* of one of the obtuse angles to meet the opposite springing line *H Q* or *H Q* prolonged perpendicularly in *C*, the line *A C* is the width of the arch-way, and the line *C H* is called the distance of obliquity; for the less *C H*, the angle *A H Q* will be nearer to a right angle, and when *C H* is nothing, the angle *A H Q* will be a right angle, and consequently *A H* will coincide with *A C*. A straight line *H R* drawn to meet the opposite side *G Q* prolonged perpendicularly in *R* is the distance between the two faces, or the distance between the front and rear elevations of the arch.

Prob. XXVII.—Given the width of the arch-way, the angle of obliquity and the distance between the two faces, to draw the plan of the arch.

Draw the straight line *A C* and make *A C* equal to the width of the arch-way. Make the angle *C A H* equal to the complement of the angle of obliquity; thus if the angle *A H C* be 50° make *C A H* equal to 40°. Draw *C H* perpendicular to *A C* and *H R* perpendicular to *A H*. Make *H R* equal to the distance between the two faces of the arch, and draw *R G* parallel to *A H* intersecting *H C* prolonged to *Q*. Draw *A G* parallel to *H Q*, and the parallelogram *A G Q H* is the plan of the aperture, *A G, H Q* being the two springing lines, *A H, G Q* the lengths of the faces.

Prob. XXVIII.—Given the length of one of the faces, the angle of obliquity, and the distance between the two faces to draw the plan of the arch-way.

Draw the straight line *A H* and make *A H* equal to the length of the given face. Make the angle *A H Q* or *A H C* equal to the angle of obliquity. Upon *A H* as a diameter, describe the semicircle *A C H* intersecting *H Q*. or *H Q* prolonged in *C*, and join *A C, C H*. Draw *H R* perpendicular to *A H*, and make *H R* equal to the distance between the two faces. Draw *G R* parallel to *A H*, and *A G* parallel to *H Q*, and *A G Q H*, is the plan of the aperture of the arch.

Prob. XXIX.—Given the width of the arch-way, the distance of obliquity, and the distance between the two faces, to draw the plan of the arch.

Draw the straight line *A C* and make *A C* equal to the width of the arch-way. Draw *C H* perpendicular to *A C*; make *C H* equal to the distance of obliquity and join *A H*. Draw *H R* perpendicular to *A H*, and make *H R* equal to the distance between the two faces Through *R* draw *G R* parallel to *A H* intersecting *H C*, or *H C* prolonged in *Q*. Draw *A G* parallel to *H Q*, and *A G Q H* is the plan.

PROB. XXX.—Given the width of the arch-way the length of one of the faces, and the distance between the two faces, to draw the plan of the arch.

Draw the straight line $A\ H$, and make $A\ H$ equal to the length of the given face. Upon $A\ H$ as a diameter describe the semicircle $A\ C\ H$. From the point A with the width of the arch-way, cut the arc in C, and join $A\ C$, $C\ H$. Draw $H\ R$ perpendicular to $A\ H$, and $H\ R$ equal to the distance between the two faces. Draw $G\ R$ parallel to $A\ H$ intersecting $H\ C$, or $H\ C$ prolonged in Q. Draw $A\ G$ parallel to $H\ Q$, and $A\ G\ Q\ H$ is the plan.

EXAMPLES FOR PRACTICE.

1. There is an oblique arch, of which the angle of obliquity is 76° 42′, the width of the arch-way 38·75 feet, the distance between the faces 26 feet, the length of one of the faces 39·82 feet; to draw the plan of this arch, when its width, its angle of obliquity, and the distance between its faces are given, and when its angle of obliquity, the length of one of its faces, and the distance between them are given, and when the width of its arch-way, the distance of its obliquity, and the distance between its faces are given,

2. There is an oblique arch of which the angle of obliquity is 50°, the length of one of the faces 60 feet, the width of the arch-way 45·96 feet, and the distance between the two faces, 28·75 feet; to draw the plan of this arch when its angle of obliquity, the length of one of its faces, and the distance between them are given, and when the width of its arch-way, the length of one of its faces and the distance between them are given.

3. There is an oblique arch of which the angle of obliquity is 26° 54′, the length of one of the faces 42 feet, the width of the arch-way 19 feet, and the distance between the two faces 14 feet; to draw the plan of this arch when its angle of obliquity, the length of one of its faces and the distance between them are given, and when the width of its arch-way, the length of one of its faces, and the distance between them are given.

It will give great facility to the student to exercise himself in drawing the plans of oblique arches to the dimensions given in the first, second, and third of these examples to a large scale, say one quarter inch to the foot, or $2\frac{1}{2}$ inches to ten feet.

Figures 1, 2, 3, in order to be contained in the plate, are drawn to a scale of which every half inch contains 10 feet; figure 1 is drawn according to the first of these proportions, figure 2 to the second, and figure 3 to the third.

Figure 1, plan of the oblique arch at Gateshead, upon the Brandling Junction Railway.

Figure 2, plan of one of the oblique arches of the bridge over the river Tees, at Croft, on the Great North of England Railway.

Figure 3, plan of the oblique arch over the river Gaunless, near Hagger Leazes Lane.

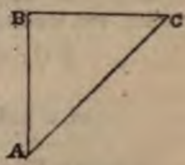
SECTION VI.

ON THE MENSURATION OF THE SIDES AND ANGLES OF A RIGHT-ANGLED TRIANGLE, TWO PARTS BEING GIVEN.

This may be divided into three cases. First, when the two sides are given, to find the third; when the two sides are given, to find the angles; and thirdly, when a side and an angle are given to find the remaining sides.

PROB. XXXI.—Given any two sides of a right-angled triangle $A\,B\,C$, to find the remaining side.

By Euclid, Prop. XLVII., Book 1, we have



$$A\,C^2 = A\,B^2 + B\,C^2$$

Or $A\,B^2 + B\,C^2 = A\,C^2$;

Therefore, $B\,C^2 = A\,C^2 - A\,B^2$

And, $A\,B^2 = A\,C^2 - B\,C^2$;

Hence, $B\,C = \sqrt{(A\,C^2 - A\,B^2)}$

And, $A\,B = \sqrt{(A\,C^2 - B\,C^2)}$

EXAMPLE.

Let $A\,C$ be equal to 42 feet, and $B\,C$ be equal to 19 feet, to find $A\,B$.

Here, $A\,B = \sqrt{(42^2 - 19^2)} = \sqrt{(1403)} = 37{\cdot}4566$.

Each of the three straight lines which comprise the surface of a triangle is called a side of that triangle. In a right-angled triangle, the two sides which contain the right angle are called the legs, of which the one is opposite to one of the acute angles, and the other adjacent to the same angle.

On the method of finding a side of a right-angled triangle, a side and an angle being given, or to find the angles when two sides are given.

The trigonometrical table contains a series of numbers amongst which will be found three numbers in the same ratio as the sides of any proposed right-angled triangle, and if two acute angles, one in the tabular triangle and the other in the proposed triangle, be equal, the sides about the equal angles will be proportional, and reciprocally, if two sides of the proposed triangle be proportional to two sides of the tabular triangle, the angles contained by these two sides of the proposed triangle shall be equal to the angle contained by the corresponding sides of the tabular triangle. The three sides of the tabular triangle are called the radius, sine, and cosine; the hypothenuse is called the radius, which, in the table of natural sines is equal to 1, or unity; therefore each of the other two sides must be less than 1. The leg of the tabular triangle, which is opposite to

an angle, is called the sine of that angle, and the leg of the tabular triangle which is adjacent to an angle, is called the cosine of that angle. The tabular triangle which is similar to a proposed triangle, will be recognised when an angle of the proposed triangle is given. For instance, let the angle of a triangle be 36°, then the two sides of the tabular triangle which contain that angle are the radius equal to 1, and the cosine equal to ·80902; therefore, when one side of the proposed triangle is given, the other can be found; and, moreover, the tabular triangle, which is similar to a proposed triangle, will be recognized when two sides of the proposed triangle are given: suppose the hypotheneuse and a leg opposite an angle to be given. In the triangle $A\ B\ C$, let the side $B\ C$ be the leg opposite the angle A and $A\ B$ the side adjacent to the angle A; now then, writing *rad*=1 for radius, *sin.* for sine, and *cos.* for cosine,

$$A\ C : B\ C :: rad. : sin.A = \frac{B\ C}{A\ C}, \text{ because rad.} = 1.$$

$$A\ C : A\ B :: rad. : cos.A = \frac{A\ B}{A\ C}$$

Therefore, the leg of a right-angled triangle which is opposite an angle divided by the hypothenuse gives the sine of that angle, and the leg of a right-angled triangle adjacent to an angle divided by the hypothenuse gives the cosine of that angle. Therefore, when two sides of a triangle are given, the angles can be found.

PROB. XXXII.—Given two sides of a right-angled triangle to find the angles by common arithmetic.

To do this by a table of natural sines, the hypothenuse must be one of the given parts, therefore if the two legs are given, the hypothenuse must be found by Prob. XXV; hence when any two of a right-angled triangle are given, the angles may be found. We shall therefore suppose the hypothenuse always to be one of the given parts.

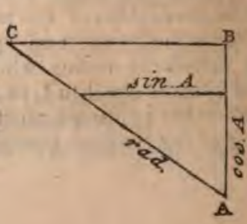
RULE.

If one of the given sides be the leg opposite the angle, divide this side by the hypothenuse and the quotient will be the sine of the angle, and if one of the given sides be the leg adjacent to the angle, divide this side by the hypothenuse, and the quotient will be the cosine of the angle as has already been shown.

EXAMPLE 1.

In the triangle $A\ B\ C$ are given the side $B\ C$ equal to 17·374 feet, and the hypothenuse $A\ C$ equal to 22·063 feet, to find the angle A, Here 17·374÷22·083=·78688.

By inspecting the table of natural sines, we find the two numbers 78676 and 78693 the first less than the sine of 51° 53′ and the second greater than the sine of 51° 54′ and since in our calculations, a minute is of very little consequence; we shall not be far wrong in calling the quantity which the angle A contains 51° 53′.



Example 2.

Given the side $A\ B$ equal to 13·63 feet, and the hypothenuse $A\ C$ equal to 22·083 to find the angle A.

Here 13·63÷22·083=·61721,

And in the table of cosines 61721 will be found to answer to 51° 53

Examples for Practice.

In the right-angled triangle $A\ B\ C$, if the hypothenuse $A\ C$ be the length of one of the faces of the arch and the angle A the angle of obliquity, the opposite leg $B\ C$ is the width of the arch-way, and the adjacent leg $A\ B$ the distance of obliquity. See the construction of the plan of an oblique arch, page xxxv and plate 10.

Therefore when the width of the arch-way and the length of one of the faces are given, the quotient obtained by dividing the width of the arch-way by the length of the given face shall be the sine of the angle of obliquity.

And the quotient obtained by dividing the distance of obliquity by the length of the given face shall be the cosine of the angle of obliquity.

1. Given the width of the arch equal to 17·374 feet, and the length of one of the faces equal to 22·083 feet to find the angle of obliquity. *Ans.* 51° 53′.

2. Given the distance of obliquity equal to 13·63 feet, and the length of one of the faces equal to 22·083 feet to find the angle of obliquity. *Ans.* 51° 53′.

3. Given the width of the arch equal to 38 75 feet, and the length of one of the faces equal to 39·82 feet, to find the angle of obliquity. *Ans.* 76° 41′.

4. Given the distance of obliquity equal to 9 feet 2 inches, and the length of one of the faces equal to 39·82 feet, to find the angle of obliquity. *Ans.* 76° 41′.

5. Given the width of the arch-way equal to 45·96 feet, and the length of one of the faces equal to 60 feet, to find the angle of obliquity *Ans.* 50°.

6. Given the distance of obliquity equal to 38·57 feet, and the length of one of the faces equal to 60 feet, to find the angle of obliquity. *Ans.* 50°.

7. Given the width of the archway equal to 19 feet, and the length of one of the faces equal to 42 feet, to find the angle of obliquity. *Ans.* 26° 54′.

8. Given the distance of obliquity equal to 37·4556 feet, and the length of one of the faces equal to 42, to find the angle of obliquity. *Ans.* 26° 54′.

9. Given the width of the arch-way equal to 22 833 feet, and the length of one of the faces equal to 27·003 feet, to find the angle of obliquity. *Ans.* 57° 44′.

10. Given the distance of obliquity equal to 14·416 feet, and the length of one of the faces equal to 27·003 feet, to find the angle of obliquity. *Ans.* 57° 44′.

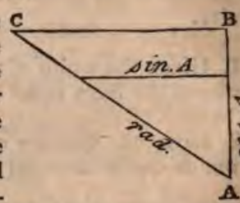
11. Given the width of the arch-way equal to 23 feet, and the length of one of the faces equal to 24.0416 feet to find the angle of obliquity. *Ans.* 73° 4′.

12. Given the distance of obliquity equal to 7·009 feet, and the length of one of the faces equal to 24·0416 feet, to find the angle of obliqity. *Ans.* 73° 4′.

13. Given the width of the arch-way equal to 17 feet, and the length of one of the faces equal to 19·781 feet, to find the angle of obliquity. *Ans.* 59° 15′.

14. Given the distance of obliquity 10·1138 feet, and the length of one of the faces equal to 19·781 feet, to find the angle of obliquity. *Ans.* 59° 15′.

PROB. XXXIII.—Given a side and an angle of a triangle, to find one of the two remaining sides,



Suppose the leg of the tabular triangle which is called the sine to be parallel to the leg of the proposed triangle which is opposite the angle, and the hypothenuse of the tabular triangle to be upon the hypothenuse of the proposed triangle, and the leg adjacent to the angle to fall upon the leg of the proposed triangle, then in the tabular triangle as the hypothenuse is called the radius, the leg opposite the angle the sine, and the leg adjacent to the angle the cosine, the radius, sine and cosine, will form a triangle similar to the hypothenuse, the base, and the leg adjacent to the angle of the proposed triangle as in the diagram.

EXAMPLE 1.

Given the angle A equal to 26° 54′ and the side $A\ C$ equal to 42 feet, to find the side $B\ C$,

Rad. : *sin.* A : : $A\ C$: $B\ C$;

or 1 : ·45243 : : 42 : $B\ C$=·45243×42=19·002 or 19 feet.

EXAMPLE 2.

Given the angle A as before and the side $A\ B$ equal to 37·455 feet, to find the side $B\ C$.

Cos. A : sin. A : : $A\ B$: $B\ C$;

or ·8918 : ·45243 : : 37·455 : $B\ C=\dfrac{\cdot 45243\times 37\cdot 455}{\cdot 8918.}$=19·001 or 19 feet nearly.

EXAMPLE 3.

Given the angle A as at first, and the side $A\ C$ equal to 42 feet to find the side $A\ B$.

Rad. : cos. A : : $A\ C$: $A\ B$;

or 1 : ·8918 : : 42 : $A\ B$=·8918+42=37·4556 feet.

Example 4.

Given the angle A as at first, and the side $B\,C$ equal to 19 feet, to find the side $A\,B$.

Sin. A : cos. A : : $B\,C$: $A\,B$.

$$\cdot 45243 \;:\; \cdot 8918 \;::\; 19 : B\,B = \frac{\cdot 8918 \times 19}{\cdot 4524} = 37\cdot 454 \text{ feet.}$$

Example 5.

Given the angle A and the side $A\,B$ equal to 37·455 feet, to find the side $A\,C$.

Cos. A : rad. : : $A\,B$: $A\,C$.

$$\cdot 8918 \;:\; 1 \;::\; 37\cdot 455 : A\,C = \frac{37\cdot 455}{\cdot 8918} - 41\cdot 99, \text{ or 42 feet nearly.}$$

Example 6.

Given the angle A as at first, and the side $B\,C$ equal to 19 feet, to find the side $A\,C$.

Sin. A : rad : : $B\,C$: $A\,C$.

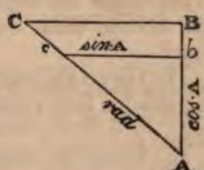
$$\cdot 45243 \;:\; 1 \;::\; 19 ; A\,C = \frac{19}{\cdot 45243} = 41\cdot 995, \text{ or 42 feet nearly.}$$

Examples for Practice.

In such of these examples where the distance between the faces and the angle of obliquity are given to find the length of the springing lines, the angle A of the triangle $A\,B\,C$ is equal to the angle of obliquity, the leg $B\,C$, opposite to A, is the distance between the two faces, and the hypothenuse $A\,C$ is the length of the springing lines or length of each abutment. See figures 1, 2, 3, plate 10.

It will therefore be,

sin. A : cos. A : : $B\,C$: $A\,B$ the distance of obliquity,
sin. A : 1 : : $B\,C$: $A\,C$ the length of each face,
1 : sin. A : : $A\,C$: $B\,C$ the width of the archway,
sin. A : 1 : : $B\,C$: $A\,C$ the length of each abutment.



1. Given the angle of obliquity equal to 51° 53′ and the width of the arch-way equal to 17·34 feet, to find the distance of obliquity.
Ans. 13·60 *feet.*

2. Given the angle of obliquity equal to 76° 42′ and the width of the archway equal to 38·75 feet to find the distance of obliquity.
Ans. 9·16 *feet.*

3. Given the angle of obliquity equal to 76°, 42′ and the distance between the two faces equal to 26 feet, to find the lengths of the two springing lines or abutments. *Ans.* 26·71 *feet.*

4. Given the angle of obliquity equal to 50°, and the length of one of the faces equal to 60 feet, to find the width of the archway ?
Ans. 45·962 *feet.*

5. Given the angle of obliquity equal to 50°, and the length of one of the faces equal to 60 feet, to find the distance of obliquity ?
Ans. 38·567 *feet.*

6. Given the angle of obliquity equal to 50°, and the distance between the two faces of the arch equal to 28·75 feet, to find the length of the abutments ? *Ans.* 37·53 *feet.*

7. Given the angle of obliquity equal to 26° 54′, and the length of one of the faces equal to 42 feet, to find the width of the archway ?
Ans. 19·002 *feet.*

8. Given the angle of obliquity equal to 26° 54′, and the length of one of the faces equal to 42 feet, to find the distance of obliquity ?
Ans. 37·4556 *feet.*

9. Given the angle of obliquity equal to 26° 54′, and the distance between the two faces of the arch equal to 14 feet, to find the length of the abutments. *Ans.* 30·944 *feet.*

10. Given the angle of obliquity equal to 57° 44′, and the length of one of the faces equal to 27·003 feet, to find the width of the archway ? *Ans.* 22·833 *feet.*

11. Given the angle of obliquity equal to 57° 44′, and the length of one of the faces equal to 27·003 feet, to find the distance of obliquity ?
Ans. 14·416 *feet.*

12. Given the angle of obliquity equal to 57° 44′, and the distance between the two faces of the arch equal to 12·5 feet, to find the length of the abutments ? *Ans.* 14·782 *feet.*

13. Given the angle of obliquity equal to 73° 4′, and the length of one of the faces equal to 24·0416 feet, to find the width of the archway ? *Ans.* 22·999 *feet.*

14. Given the angle of obliquity equal to 73° 4′, and the length of one of the faces equal to 24·0416 feet, to find the distance of obliquity ?
Ans. 7·002356 *feet.*

15. Given the angle of obliquity equal to 73° 4′, and the distance between the two faces equal to 12·1666 feet, to find the length of the abutments or springing lines ? *Ans.* 12·7191 *feet.*

16. Given the angle of obliquity equal to 59° 15′, and the length of one of the faces equal to 19·781 feet, to find the width of the archway ? *Ans.* 16·9999 *feet.*

17. Given the angle of obliquity equal to 59° 15′, and the length of the oblique face equal to 19·781 feet, to find the distance of obliquity ?
Ans. 10·1138 *feet.*

18 Given the angle of obliquity equal to 59° 15′, and the length of each abutment equal to 30·875 feet, to find the distance between the two faces ? *Ans.* 26·534 *feet.*

SECTION VII.

ON THE USE OF THE OBLIQUE-ANGLED TRIANGLE.

PROB. XXXIV.—To find a point in the side of a triangle, in which a perpendicular drawn from the opposite angle meets that side.

Let $B\ C=a$, $A\ C=b$, and $A\ B=c$; also let $B\ D=x$, and $C\ D=y$; then will $A\ D=c-x$.

By Euclid, Book 1., Proposition XLVII.

$$y^2=a^2-x^2$$
$$y^2=b^2-(c-x)^2$$

By equality,

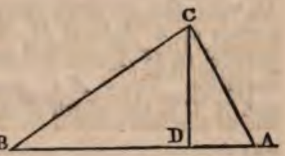
$$b^2-(c-x)^2=a^2-x^2$$

Or, $b^2-c^2+2cx-x^2=a^2-x^2$

Or, $b^2-c^2+2cx=a^2$*

$$2cx-c^2=a^2-b^2$$

Or, $c(2x-c)=(a+b)(a-b)$



Hence, by converting the equation into a proportion, we get

$$c : a+b :: a-b : 2x-c$$

Now since $x=B\ D$ one of the segments of the base, and $c-x=A\ D$ the other segment: if x be the greater segment, $c-x$ is the less; and $c-x$ subtracted from x is $x-(c-x)=2x-c$; the difference of the segments of the base; therefore, it will be

As the base or longest side of a triangle,
Is to the sum of the other two sides;
So is the difference of these two sides
To the difference of the segments of the base.

By this rule the distance of the perpendicular upon the base from one of its ends will be easily found; for, having found the difference of the bases of the two right-angled triangles; add half the difference to half the longest side AB, of the triangle $A\ B\ C$, the sum will be the length of $B\ D$, the greater segment, and subtract half the difference from half the base $A\ B$, the difference will be the length $A\ D$ of the less segment.

---

*Since $b^2-c^2+2cx=a^2$ by transposition, $2cx=a^2+c^2-b^2$ hence $x=\dfrac{a^2+c^2-b^2}{2c}$; but in the right-angled triangle $B\ D\ C$, right-angled at D, making $B\ C=a$, radius; then $B\ D=x$ shall be the cosine of the angle B;

Hence, radius$=1$: cos. B :: a : $x=a$ cos. B.

Hence, a cos. $B=\dfrac{a^2+c^2-b^2}{2c}$

Hence, cos. $B=\dfrac{a^2+c^2-b^2}{2ac}$

PROB. XXXV.—Having executed the piers of an oblique arch to the height of the springers; to draw the plan of the archway from the measures then taken,

EXAMPLE 1.

Given the adjacent sides *H Q*, *H A*, (fig. 1,) of the parallelogram *H A G Q*, and the diagonal *A Q* respectively, equal to 32ft. 6in., 22ft. 1in., and 25ft. $7\frac{1}{2}$in.; to draw a plan of the oblique arch, and to find the width of the aperture, *H A* being the length of one of the faces, and *H Q* the length of one of the abutments.

Draw the straight line *H Q*, and make *H Q* equal to 32ft. 6in. From *H*, with the distance 22ft. 1in., describe an arc at *A*, and from *Q*, with the distance 25ft. $7\frac{1}{2}$in., describe another arc, cutting the former at *A*. Join *A H*, *A Q*; draw *A G* parallel to *H Q*, and *Q G* parallel to *H A*, and the parallelogram *A H Q G* is the plan of the aperture. Draw *A C* perpendicular to *H Q*, meeting *H Q* in *C*, and *A C* is the width of the archway.

EXAMPLE 2.

Given *A H* (fig. 2,) equal to 19ft $9\frac{7}{8}$in., and *H Q* equal to 30ft. $10\frac{1}{2}$in., and the perpendicular *A C*, which isthe width of the arch, equal to 17ft., to draw the plan, *A H* being the length of one of the faces, and *H Q* the length of one of the abutments, and the angle *A H Q* being the angle of obliquity.

Draw the straight line *A H*, and make *A H* equal to 19ft. $9\frac{7}{8}$in. Upon *A H*, as a diameter, describe the semicircle *A C H*. From *A*, with the distance of 17ft., cut the semicircular arc in *C*, and join *A C*, *C H*. Prolong *H C* to *Q*, and make *H Q* equal to 30ft. $10\frac{1}{2}$in. Draw *A G* parallel to *H Q*, and *Q G* parallel to *H A*, and the parallelogram *A H Q G* is the plan.

EXAMPLE 3.

In the parallelogram *H A G Q*, (fig. 3,) which represents the plan of the aperture of an oblique arch, the abutment *H Q* measures 35ft. 6in.; the perpendicular distance between the abutments measures 28ft., and the distance of obliquity 9ft 8in; to draw the plan of the arch.

Draw the straight line *A C*, and make *A C* equal to 28ft. Draw *C H* perpendicular to *A C*, and make *C H* equal to 9ft. 8in. Prolong *H C* to Q, and make *H Q* equal to 35ft. 6in. Join *A H*; draw *A G* parallel to *H Q*, and *Q G* parallel to *A H*, and *H A G Q* is the plan of the aperture or archway required.

Observations.

The dimensions measured upon the side of the parallelogram which is to be one of the faces of the arch, and along one of the springing lines of one of the abutments and the perpendicular distance between the abutment edges as given in example 2 are the most convenient for construction and require very little calculation. A plan drawn from the measures taken as in example 3, is also easily constructed and requires no calculation; but the distance of obliquity must be previously found by a perpendicular. The dimensions taken as in figure 1, are not only inconvenient with regard to the position of the plan but require considerable calculation as shown on the following

Calculation of the parts of Ex. 1, from prop. xxxiv.

The base $H\ Q$ of the triangle $H\ A\ Q$, being...32ft. 6 in.=32·5
One of the other two sides, $Q\ A$,.............. . 25ft. $7\frac{1}{2}$in.=25·62
And the third side $H\ A$, ...22ft. 1 in.=22·083
The sum of these two sides is .. ..47ft. $8\frac{1}{2}$in.=47·708
And their difference .... 3ft. $6\frac{1}{2}$in.=3·542

Hence,

$$32{\cdot}5 : 47{\cdot}708 :: 3{\cdot}542 : \frac{47{\cdot}708 \times 3{\cdot}542}{32{\cdot}5} = 5{\cdot}2$$ which is the difference of the segments of the base, very nearly.

Now half the base is the half of ....32·5=16.25
And half the diff. of the segments is the half of ...5·2= 2·6
Hence the greater segment ....=18·85
And the less ....=13·65
And $A\ C = \sqrt{(A\ H^2 - H\ C^2)} = \sqrt{(22{\cdot}083^2 - 13{\cdot}65^2)} = 17{\cdot}35$ feet.

Calculation for $C\ H$, Ex. 2.

$C\ H = \sqrt{(A\ H^2 - A\ C^2)} = \sqrt{(14968{\cdot}515625)} = 122{\cdot}345$in. 10·195 feet.

SECTION VIII.

ON THE RADIUS OF CURVATURE.

PROB. XXXIV.—Given the radius of the surface of a cylinder, and the angle of inclination which the developement of a spiral line on that surface makes with a line drawn on the developement parallel to the axis of the cylinder; to find the radius of curvature of the spiral.

Draw the straight line $U\ V$ (fig. 1,) to represent the axis of the cylinder. In $U\ V$ take any point, I, and through I draw $G\ H$ perpendicular to $U\ V$. Make $I\ G$, $I\ H$, each equal to the radius of the cylinder. Parallel to $U\ V$ through G draw $Q\ R$, and through H draw $S\ T$; then $Q\ R\ T\ S$ will be a plane passing through the axis of the cylinder. From the point I as a centre, with the radius $I\ H$ or $I\ G$ describe the semicircle $H\ B\ G$. In the triangle $A\ D\ F$ draw $D\ F$ parallel to $U\ V$, and $A\ D$ perpendicular to $D\ F$ Take the height $D\ F$ at pleasure. and make the angle $D\ F\ A$ equal to the angle which the spiral line makes with a straight line on the surface of the cylinder, or with a line on the surface of the cylinder parallel to its axis. Draw $G\ L$ parallel to $A\ F$, meeting $Q\ R$ in G, and $S\ T$ in L, intersecting $U\ V$ in J. Draw $J\ K$ perpendicular to $G\ L$, and make $J\ K$ equal to $I\ H$ or $I\ G$ the radius of the cylinder. Upon the semi-axis major $G\ L$, and with the semi-axis minor $J\ K$, describe the semi-elipse $G\ K\ L$ which shall be a section of the cylinder parallel to $A\ F$ the developement of the spiral line, and which shall have the same radius of curvature at K the extremity of the axis minor, which the spiral has at the same point. Now, in the triangle $A\ D\ F$ let $D\ F = a$, $A\ D = b$, and let the hypothenuse $A\ F = h$; then $h^2 = a^2 + b^2$; moreover let the radius of the cylinder be denoted by r,

Then by similar triangles, $D\ A\ F$, $I\ G\ J$.

$$A\ D : A\ F :: G\ I : G\ J$$

Or $b : h :: r : G\ J = \frac{rh}{b}$ which is the length of the semi-axis major; but from the property of the ellipse

$$K\ J : G\ J :: G\ J : \text{to the radius of curvature at } K.$$

Or, $\quad r : \frac{rh}{b} :: \frac{rh}{b} : \frac{rh^2}{b^2} = \frac{r(a^2+b^2)}{b^2}$ which is the radius of curvature at K.

Now the hypothenuse $A\ F$, of the triangle $A\ D\ F$, is the developement or length of a spiral line, the base $A\ D = b$, a portion of the circumference of the cylinder, in a plane perpendicular to the axis, equal to the length of the arc $A\ B\ C$, passing from one extremity of the spiral, and $D\ F = a$ the length of a line on the curved surfaee of the cylinder, parallel to the axis drawn from the other extremity of the spiral, meeting the plane of the circle perpendicularly in the other extremity of the arc, hence we have the following

RULE.

Multiply the square of the length of the spiral line by the radius of the cylinder, divide the product by the square of the length of the arc, and the quotient is the radius of curvature.

Example.

Given the radius of the well-hole of a spiral stair equal to 2 feet 3 inches, the height of a step equal to 6 inches and the breadth of a step equal to 4 inches on the circumference, required the radius of curvature of the rail, and of the string-board under the steps.

The height of six steps is 3 feet, and the length of six steps round the arc upon which the cylindric surface of the well-hole stands is 2 feet; moreover 2 feet 3 inches$=2{\cdot}25$, the radius of the well-hole.

Now $2^2=4$ the square of the length of the arc,
$3^2=9$ square of the height,
Sum $=13$ square of the length of the spriral,
$13\times 2{\cdot}25=29{\cdot}25$ prod. of rad. and square of length of spiral,
$29{\cdot}25\div 4=7{\cdot}3125$ feet, the radius of curvature required.

Again, let $A\ B\ C$ (Fig. 2) be a section of the cylinder in a plane perpendicular to the axis, $A\ C$ being the chord and $a\ B$ the height of the arc, and I the centre of the circle. Also, let $H\ G$ be drawn through I parallel to $A\ C$, and $U\ V$ through I, perpendicular to $H\ G$, to represent the axis of the cylinder. Let $A\ D$ be drawn parallel to $A\ C$, making $A\ D$ equal to the length of the arc $A\ B\ C$. Draw $D\ E$ perpendicular to $A\ D$; make $D\ E$ equal to the distance of obliquity, and join $A\ E$. Draw $A\ F$ perpendicular to $A\ E$, and prolong $E\ D$ to F. Then $A\ E$ will be the developement or length of a spiral line in the face parallel and equal to the lines which are the developements of the spirals of the joint lines, and $A\ F$ will be parallel and equal to the lines which are the developements of the spirals of the bed lines. Let the semicircle $H\ A\ B\ C\ G$ be completed; draw $H\ M$ and $H\ L$ perpendicular to $H\ G$. Moreover, draw $G\ M$, $G\ L$ respectively parallel to $A\ E$, $A\ F$. Then $G\ M$ is the axis major of an ellipse, of which the radius of curvature at the extremity of the axis minor would be that of the spirals upon which the ends meet in the intrados, and $G\ L$, the axis major of the ellipse, of which the radius of curvature at the extremity of the axis minor would be that of the spiral bed lines.

Then because $E\ A\ F$ is a right-angled triangle, right-angled at the vertex A, and $A\ D$ is drawn to meet the hypothenuse $E\ F$ perpendicularly in D, the triangle $E\ A\ F$ is divided into two right-angled triangles, which are similar to one another and to the whole triangle $E\ A\ F$; therefore, the sides about the equal angles are proportional, and the homologous sides are opposite equal angles.

Let $c=A\ D$ the length of the arc $A\ B\ C$ and $d=D\ E$ the distance of obliquity; also let $h=A\ E$; then $h^2=c^2+d^2$; moreover, let $r=P\ N=J\ K=I\ H$, the radius of the cylinder.

By similar triangles $D\ A\ E$ and $I\ G\ P$.

$$D A : A E :: I G : G P.$$

or, $c . h :: r : G P=\dfrac{r\,h}{c}$ the semi-axes major of the ellipse $G\ N\ M$.

From the property of the ellipse and the radius of curvature
$PN : PG :: PG$: the radius of curvature at N.

or, $r : \frac{rh}{c} :: \frac{rh}{c} : \frac{rh^2}{c^2} = \frac{r(c^2+d^2)}{c^2}$ which is the radius of curvature of the spiral lines of the joints, and is equal to that of the ellipse $M\,N\,G$ at the extremity N of the axis minor.

Again, by similar triangles $D\,E\,A$, or $D\,A\,F$, and $I\,G\,J$.
$DE : EA :: IG : GJ$.

or, $d : h :: r : GJ = \frac{rh}{d}$ the semi-axis major of the ellipse $G\,K\,L$, and from the property of the ellipse and the radius of curvature.

$KJ : JG :: JG$: the radius of curvature at K.

or, $r : \frac{rh}{d} :: \frac{rh}{d} : \frac{rh^2}{d^2} = \frac{r(c^2+d^2)}{d^2}$ which is the radius of curvature of the ellipse $G\,K\,L$ at the extremity K of the axis minor, and is equal to that of the spiral bed-lines.

RULE.

Divide the product of the radius of the cylinder and thesquare of the length of the spiral joint lines by the square of the circular arc and the quotient shall be the radius of curvature of the spiral joint lines, and the same product divided by the square of the distance of obliquity shall be the radius of curvature of the spiral bed-lines.

EXAMPLE.

Required the radius of curvature of each series of spiral lines for the edges of the beds and the lower edges of the joints, given the chord equal to 38·75 feet, the height of the arc equal to 6 feet, and the distance of obliquity equal to $9\frac{1}{6}$ feet.

Here the radius of the cylinder is 34·2825 feet, see the first of the practical examples prob. xxv. As the length of a stone will only admit of a very small portion of the curve however great the radius may be; the radius being a little more or less in a great length will not affect the accuracy of the work; in practice it is therefore not necessary to retain more than two decimal parts; hence, calling 34·2825 only 34·28, and 41·18156 only 41·18; then proceeding according to the rule.

$AD^2 = 41{\cdot}18^2 = 1695{\cdot}7924$

$DE^2 = 55^2 \div 6^2 = 84{\cdot}0777$

AE^2 the sum $= 1779{\cdot}8701$ the square of the spiral joint lines.

$34{\cdot}28 \times 1779{\cdot}8701 = 61013{\cdot}947028$

$\frac{61013{\cdot}9470}{84{\cdot}0777} = 725{\cdot}6$ ft. radius of curvature of bed line spirals.

$\frac{61013{\cdot}9470}{1695{\cdot}7924} = 35{\cdot}9$ ft., rad. of curvature of joint line spirals.

SECTION IX.

ON THE ANGLE OF THE TWIST.

PROB. XXXV.—To find the angle of the twist of two spiral lines in two given cylindric surfaces and in the same spiral surface, given the angle which the spiral in the inner cylindric surface makes with a plane perpendicular to the axis of the cylinder.

Let *A B C* (Fig. 1.) be a right section of the concave surface, the chord *A C*, being 30 feet, and the height of the arc 10 feet, and let the concentric arc *D E F* be a right section of the convex surface which contains the other spiral at 4 feet distant from the arc, *A B C*; that is, whatever be the radius of the arc *A B C*, the radius of the arc *D E F* will be 4 feet more; let *g c a* be the angle which the spiral in the concave surface, makes with its base, which is equal to the length of the arc *A B C*, and let the angle *a c g* of the triangle *g c a* be 40°.

The length of the arc *A B C* to a chord of 30 feet, and height 10 feet, will be found by Prob. xxiv, page xxxi, to be 38·22 feet, and the radius of the arc of the same dimensions by Prob. xxv, page xxxiii, to be 16·25 feet; therefore, 16·25+4·=20·25 the radius of the convex surface; and because the arcs of similar segments are as their radii; hence

16·25 : 20·25 : : 38·22 : the length of the arc *D E F*, which being found is 47·628.

In the triangle *g a c* of the developement of the spiral upon the coucave surface make *a c* equal to 38·22 feet, the angle *a c g* equal to 40°; then, by the following statement,

rad. : tan. 40° : : 38·22 : *a g*=32·07 feet, nearly.

In the right-angled triangle *g a f*, we have the two sides *a f*, *a g*, respectively equal to 47·628 feet, and 32·07 feet, to find the angle *a f g*,

Therefore, 47·628 : 32·07 . : rad. : tan. of the angle *a f g*.

Hence, the angle *a f g* will be found to be about 33° 57′

But without finding the lengths of the arcs we might have arrived at the same conclusion by using the radii, which are proportional to the arcs. The radius *I A* of the concave surface being found as above is 16·25 feet and the radius *I D* of the convex surface is 16·25+4=20·25 feet.

In the triangle *g a c* we have the angle *a c g*=40° and the side *a c*=16·25, to find the side *a g*, which by trigonometry will be found to be 13·63 feet; thus,

rad : tan. 40° : : 16·25 : *ac*=13·63.

And in the right angled triangle *g a f* we have the side *a f*=20·25, and the side *a g*=13·63, to find the angle *a f g*, which will be found to be 33° 57′, as before; thus,

20·25 : 13·63 : : rad : tan. 33° 5′.

Now ∠ *a g f*—∠ *a g c*=∠ *a c g*—∠ *a f g*,

Or 40°—33° 57′=6° 3′ the angle of the twist.

TO FIND THE ANGLE OF THE TWIST IN PRACTICE.

It will be sufficient to divide the arcs $A\ B\ C$, $D\ E\ F$, each into the same number of equal parts, as 8, and the sum of the chords will be very nearly equal to the length of each respective arc. In the right-angled triangle $c\ a\ g$, make the base $a\ c$ equal to the whole of the 8 parts of the arc $A\ B\ C$, or any number of them, and draw $a\ g$ perpendicular to $a\ c$. Draw $c\ g$, making the angle $a\ c\ g$ equal to 40°; prolong $a\ c$ to f, and make $a\ f$ equal to the whole of the 8 parts of the arc $D\ E\ F$, or to the same number of them that $a\ c$ is of the arc $A\ B\ C$. Draw $f\ g$, and the angle $c\ g\ f$ is the angle of the twist.

Or thus; draw the straight line $X\ Z$ (fig. 2) and $X\ W$ perpendicular to $X\ Z$. In $X\ Z$ make $X\ Y$ equal to $I\ A$ or $I\ C$ the radius of the concave surface, and make $X\ Z$ equal to $I\ D$ or $I\ F$ the radius of the convex surface; and make the angle $X\ Y\ W$ equal to 40°. Join $Z\ W$, and the angle $Y\ W\ Z$ is the angle of the twist.

For the arcs of similar sigments are as the radii, thus $I\ A : I\ D$: arc $A\ B\ C$: arc $D\ E\ F$.

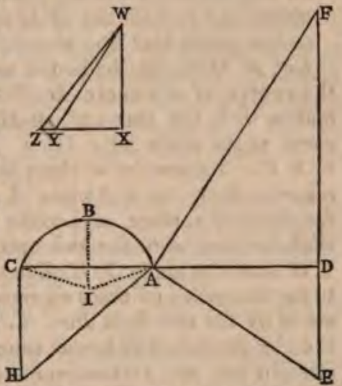
If $A\ B\ C\ F\ E\ D$ be considered as the plan of a winding stair, $a\ c$ the stretch out of the ends of the steps round $A\ B\ C$; and $a\ f$ the stretch out of the ends of the steps round $D\ E\ F$; and if the angle $a\ c\ g$ be the inclination of the spiral which passes through the nossings in the cylindric surface over the arc $A\ B\ C$, the angle $a\ f\ g$ shall be the inclination of the steps in the cylindric surface which stands upon the arc $D\ E\ F$. The angle $c\ g\ f$, which is the difference of the angles $a\ g\ f$ and $a\ g\ c$, shall be the angle of the twist which will be found necessary in working the soffit of the stair to a spiral surface, observing that the ends of a single step are in the same proportion as the triangles $g\ a\ c$, $g\ a\ f$, and differ from a plane surface by an angle equal to $c\ g\ f$. If the inclination of the steps begin with the line $A\ D$, the stair is said to be right-handed.

This principle is the most elegible in constructing the spiral pump attributed to Archimedes.

In the step of a right-hand stair properly prepared, the workman will find, by placing himself adjacent to the concave end, the convex end to be sunk from the right to the left side of the stone in a straight line, so that this line will make an angle below the plane which touches the curve line of the upper edge of the concave end equal to the angle of the twist.

And in the step of a left-hand stair properly prepared, the workman will find, by placing himself adjacent to the concave end, the convex end to be sunk from the left to the right side of the stone in a straight line, so that this line will make an angle below the plane which touches the curve line of the upper edge of the concave equal to the angle of the twist; thus, No. 1 shows the ends of the step of a right-hand stair, No. 2, the ends of the step of a left-hand stair.

PROB. XXXVI.—In an oblique arch are given, the chord of the right section of the concave surface equal to 20 feet, the he igh o he arc equal 7 feet, the distance of obliquity equal to 16 feet, and the breadth of the beds equal to 18 inches or 1ft. 6 in.; to find the angle of the twist.



Draw the straight line *C D*, and in *C D* make *A C* equal to 20 feet, and upon *A C* describe the arc of a circle of which the height is seven feet. Make *A D* equal to the length of the arc *A B C*, through *C* draw *C H*, and through *D* draw *F E* perpendicular to *C D*. Make *C H* and *D E* each equal to 16 feet, which is the distance of obliquity, and join *A H* and *A E*. Draw *A F* perpendicular to *A E*, and the angle *D A F*, will then be equal to the angle *D E A*. Draw *X Y* parallel to *A D*, and make *X Y* equal to *I A* or *I C*, the radius of the arc *A B C* which is the section of the concave surface. Draw *X W* perpendicular to *X Y*, and draw *Y W* parallel to *A F*. Prolong *X Y* to *Z*, and make *Y Z* equal to 18 in.; therefore *X Z* will be equal to the radius of the convex surface. Join *Z W*, and the angle *Z W Y* shall be equal to the angle of the twist, or by calculation, thus

From the chord equal to 20, and height of the arc equal to 7, will be found (Prob. xxiv, page xxxi), 25·9994=26 nearly, for the length of the arc *A B*, and from the same dimensions will be found, (Prob. xxv, page xxxiii), 10·642 the radius of the arc or of the canvave surface, In the triangle *A D E*, right angled at *D* are given the side *A D* equal to 26, and the side *D E* equal to 16, from which will be found the angle *D E A* equal to 49° 18′, thus

$$D E : A D :: \text{rad} : \text{tan. } D E A$$

$$\text{or} \quad 16 : 26 :: 1 : \text{tan. } D E A = \text{tan. } 49^\circ 18' = \angle X Y W.$$

In the triangle *W X Y* right angled at *X* are given the angle *X Y W* equal to 49° 18′, and the side *X Y* equal to 10·642, from which will be found the perpendicular *W X* equal to 17·293, thus

$$\text{rad} : \text{tan. } X Y W :: Y X : W X$$

$$\text{or} \quad 1 : 1{\cdot}6261 :: 10{\cdot}642 : W X = 17{\cdot}293$$

In the right angled triangle *W X Z*, we have $X Z = 10{\cdot}642 + 1{\cdot}5 = 12{\cdot}142$; hence from the two sides *W X*, *Z X* respectively equal to 17·293, and 12·142, will be found the angle *X Z W* equal to 54°56′, thus

$$Z X : W X :: \text{rad} : \text{tan. } W Z X$$

$$\text{or} \quad 12{\cdot}142 : 17{\cdot}293 :: 1 : \text{tan. } 54^\circ 56'$$

$$\text{Hence } \angle X Y W - \angle X Z W = Y W Z$$

or $54^\circ 56' - 49^\circ 18' = 5^\circ 38'$ which is the angle of the twist.

[illegible] of a semi- [illegible] perpendicularly [illegible] the angle of [illegible] of section [illegible] point [illegible] and S [illegible] the axis [illegible] the [illegible] parallel [illegible] to be [illegible] perpen- [illegible] line AJ [illegible] cylinder.

[illegible] considered [illegible] angle [illegible] angle of [illegible] cylinder in the [illegible] and [illegible] angle required. [illegible] faces [illegible] face [illegible]

[illegible] line AF draw GH [illegible] and draw [illegible] make GO [illegible] angle [illegible] curved sur- [illegible]

[illegible] curve, the [illegible] nothing: [illegible] the plane [illegible] side of [illegible] P, so that [illegible] ement of [illegible] from R as [illegible]

[illegible] the heads [illegible] the sides [illegible] line, which must be a tangent [illegible] the two lines meet. This will be shown in its proper place.

---

*Join AR, and bisect AR in m; draw R n perpendicular to SR. Join Sm which prolong to n, and join nA, and nA, or nA prolongued is a tangent to the ellipse at the point A.

SECTION XI.

OBLIQUE ARCH WITH PLANE JOINTS.

An oblique arch is a construction of masonry upon the intersection of two ways which cross each other at oblique angles.

An oblique arch with plane joints, is that in which the beds of the stones are in planes, passing through the axis of the cylinder.

The right section of an oblique arch, is a section perpendicular to the axis of the cylinder which forms the centre for turning the arch.

The right section of an oblique arch with plane joints, exhibits the arc of a circle divided into as many equal parts as there are courses of stones, by straight lines which radiate to the centre of the arc, the straight lines being the sections of the beds, and the arc a section of the intrados or cylindric surface.

The intrados, or underside of every arch here treated of, is the concave surface of a cylinder comprised between two planes which exhibit the two elevations of the arch. These planes are generally perpendicular to the horizon, and parallel to each other.

Oblique arches are of two kinds, one with plane joints and the other with spiral joints.

In the oblique arch with plane joints, the planes of the joints being parallel to the axis, intersect each face of the arch in very oblique angles, and only one of the joints can be perpendicular to the face, and this is when each elevation is a semi-ellipse, and when the plane of a joint intersects the figure in a straight line coinciding with the axis major. All the other joints, as they recede from the centre, are more and more oblique till they reach the summit of the arch. As every oblique joint causes the dihedral angles made by the face and that joint to be very unequal, the obtuse dihedral angle will be much stronger than that which is acute, these angles being supplements of each other. Therefore oblique arches with plane joints should never be used where great strength is necessary, and where the angle of obliquity is very acute, the oblique arch with spiral joints should only be employed, as the spiral joints are as nearly perpendiculur to the face as the construction will admit of. For in order to have the stones of one identical form, or such as would fill the same mould were they of one length, it is necessary that the developement of the spirals which form the bed lines should be parallel straight lines, of which two of them will be perpendicular to the face of the arch, and the others nearly so.

The cylinder here referred to is not that of a simple cylinder, but of that description which mathematicians call a hollow cylinder consisting of two concentric surfaces of which the interior is concave and the exterior convex; these surfaces without regarding the distinction of concave and convex are called by the name of cylindric surfaces: though concave and convex surfaces apply to many other solids as well as the cylinder; but as our object is the oblique arch, and as the cylinder only applies to the oblique arch wherever concave or convex is used, it must be understood to be the concave or convex surface of a cylinder.

OBLIQUE ARCH WITH PLANE JOINTS.

Prob. XXXVIII.—Given the elevation and plan of a semi-elliptic arch, which is the oblique section of a cylinder of which the right section of the intrados is a semi-circle; to find the angles for cutting the quoin heads.

Let No. 1 be a plan of the arch of which the angle $U\ V\ W$ is that of the acute-angled abutment; and let No. 2 be the elevation in which the lines $b\ h$, $c\ i$, $d\ j$, &c., represent the joints of the stones in the face, meeting the inner curve in the points b, c, d, &c., and the outer curve in the points h, i. j., &c.

Draw $\alpha\ \beta$ parallel to $V\ Y$, at such a distance from $V\ Y$ as the plane of the rear elevation may be supposed to be distant from the plane of the front, meeting $V\ W$ in α, and $Y\ Z$ in β, and the parallelogram $\alpha\ V\ Y\ \beta$ shall be the plan of the aperture. Draw $V\ \gamma$ perpendicular to $V\ Y$, meeting $\alpha\ \beta$ in γ. Draw $b\ g$, $c\ l$, $d\ n$, &c., parallel to $U\ V$, or $g\ u$; make $b\ g$, $c\ l$, $d\ n$, &c., each equal to $\alpha\ \gamma$, and complete the parallelograms $g\ b\ h\ l$, $l\ c\ i\ m$, $n\ d\ j\ o$, &c., which shall represent the beds of the stones.

Now observing that each of the parallel lines $b\ g$, $c\ l$, $d\ n$, &c., is the projection of a joint line in the intrados of the arch, and forms an angle with the face of the arch in a plane perpendicular to that face, equal to the angle $U\ V\ W$ of the acute abutment; it is, therefore, evident that at every joint will be formed a right trihedral, of which the angle of one of the right faces is equal to the acute angle of the plan, and the other in the plane of the face equal to the angle made by the joint line and a line parallel to the base, and that the angle of the oblique face is the angle made by the two joint lines, the one being in the face and the other in the intrados.

To construct the trihedral for any one of the joints as $b\ h$ in No. 3 or in No. 4, draw $A\ K$ parallel to $b\ h$, and $A\ F$ parallel to $b\ g$; and make the angle $F\ A\ J$ equal to the angle $U\ V\ W$. Then $F\ A\ K$ is the angle of the adjacent face, and $F\ A\ J$ the angle of the opposite face. Proceed now as in Prob. xxi, page xxvii, thus

From any convenient point G in the right edge $A\ F$, draw $G\ I$, intersecting the adjacent edge $A\ K$ perpendicularly in H, and draw $G\ E$ perpendicular to $A\ F$, meeting $A\ J$ in E. From the point A, with the distance $A\ E$ cut $H\ I$ in I; through I draw $A\ L$, and the angle $K\ A\ L$ or $H\ A\ I$ shall be the angle of the oblique face, that is the angle made by the joint line on the intrados and the joint line on the face.

In $A\ F$ make $G\ O$ equal to $G\ H$; join $O\ E$, and the angle $G\ O\ E$ is the dihedral angle, or the angle made by the bed and face of the stone.

Or the angles of the beds and the dihedral angles of the faces and beds may be found in one diagram in the following manner:—

In fig. 2 draw $P\,Q$ parallel to $A\,F$ or $V\,Y$, and in $P\,Q$ take the point L, and draw $L\,H$, $L\,I$, $L\,J$, &c., respectively, parallel to the joint lines $b\,h$, $c\,i$, $d\,j$, &c., in the elevation No. 2; and towards P take the distance $L\,G$ of any convenient length, and draw $G\,H$, $G\,I$, $G\,J$, &c., respectively perpendicular to $L\,H$, $L\,I$, $L\,J$, &c In $G\,L$ make $G\,a$, $G\,b$, $G\,c$, &c., respectively equal to $G\,H$, $G\,I$, $G\,J$. Make the angle $G\,L\,M$ equal to the acute angle which the axis of the cylinder makes with the line of section of the elevation, or equal to the angle of the acute abutment, viz., equal to $U\,V\,W$, and draw $G\,M$ perpendicular to $P\,Q$. Join $a\,M$, $b\,M$, $c\,M$, &c., and the angles $P\,a\,M$, $P\,b\,M$, $P\,c\,M$, &c., shall be the dihedral angles made by the beds and faces at the joints $b\,h$, $c\,i$, $d\,j$, &c. From L with the radius $L\,M$, describe the arc $M\,z\,y\,x$. In $L\,Q$ make $L\,u$, $L\,v$, $L\,w$, &c., respectively equal to $L\,H$, $L\,I$, $L\,J$, &c. Perpendicular to $P\,Q$ draw $u\,x$, $v\,y$, $w\,z$, &c., and the angles $Q\,L\,x$, $Q\,L\,y$, $Q\,L\,z$, &c., are the angles of the beds. To avoid confusion the work is placed below the line $P\,Q$ instead of being above.

Remarks.

In the construction of the oblique arch with plane joints, the arch stones which are supported by the acute angled abutment have their dihedral angles adjacent to the upper beds as they rise towards the middle decreasing. The first stone has its dihedral angle equal to a right-angle, and if a stone be supposed to have a joint in the summit of the arch, the dihedral angle of that stone would be equal to the acute angle of the abutment. Also the angles of the beds of the stones which rise from the said acute angled abutment are on the contrary continually increasing, as they ascend towards the crown, the lower bed of the first stone being the same as the acute angled abutment upon which it is placed; and if a joint be supposed to be at the summit, the angle of the bed of the adjacent stone would be a right angle.

With regard to the dihedral angles of the arch stones which rise from the obtuse angled abutment, those of the upper beds are continually inereasing, as they asceud towards the summit or crown, the dihedral angle of the first stone at the bottom bed being a right angle, and that at the summit equal to the angle of the obtuse abutment. Also the angles of the beds as they rise towards the crown are, on the contrary, continually decreasing from an angle equal to that of the abutment, to a right-angle at the crown.

On both sides of the arch the dihedral angles of the under beds are the supplements of those of the upper beds, and the angles of the beds which rise from the obtuse angled abutment are the supplements of the angles of the beds of the stones which rise from the acute angled abutment.

The position of the joints in plate 14 is not confined; in plate 15 and 16 they are regulated by joints of the right section, and are found as in Prob. XIV., page 17. In plates 15 and 16, *L H, L I, L J., &c.*, are the continuations of the joints to the centre *L*. In plate 16 the elevation answers to a right section, which is the segment of a circle. The method of finding the dihedral angles, as also the angles of the beds, will be found as before explained, thus:—

Draw *G H, G I, G J*, respectively perpendicular to *L H, L I, L J. &c.* In *G L* make *G a, G b, G c, &c.*, respectively equal to *G H, G I, G J, &c.* Make the angle *G L M* equal to the acute angle which the axis of the cylinder makes with the line of the elevation, or equal to the angle of the acute abutment, viz., equal to *V W X*, and draw *G M* perpendicular to *G L*. Join *a M, b M, c M, &c.*, and the angles *P a M, P b M, P c M, &c.*, shall be the dihedral angles at the joints which radiate to the centre *L*. From *L*, with the radius *L M* describe the arc *M z y x*. In *L Q* make *L u, L v, L w, &c.*, respectively equal to *L H, L I, L J, &c* Perpendicular to *G L* draw *u x, v y. w z, &c.*, and *Q L x, Q L y, Q L z, &c.*, are the angles of the beds.

A

TREATISE

ON

THE OBLIQUE ARCH.

ON THE OBLIQUE ARCH, WITH SPIRAL JOINTS.

If a straight line move perpendicularly upon another straight line at rest, in a fixed plane, in such a manner that if the distance passed over upon the fixed line by one extremity of the moving line, be proportional to the angle which the moving line makes with the plane, the other extremity will describe a curve on the surface of a cylinder *called a cylindric spiral.*

From this definition of a spiral line it is evident that the radius of the cylinder is equal to the length of the line which describes the spiral.

A surface, described by a straight line, moving perpendicularly upon another straight line at rest, in such a manner that the distance passed over upon the fixed line, by one extremity of the moving line, may be proportional to the angle which the describing line makes with a fixed plane, is called *a spiral surface,* and the fixed line is called *the axis of the spiral surface.*

If a spiral surface be cut by a cylindric surface, having the same axis as the spiral surface, the cylindric section of the spiral surface is a spiral line.

An oblique arch, with spiral joints, is that in which the surfaces of the beds and the surfaces of the joints are both spiral surfaces.

If an oblique arch, with spiral joints, be executed accordingto the principles here established, and cut by a plane perpendicular to the axis of the cylinder, the section will exhibit a series of straight lines, dividing the arc of a circle into smaller arcs, and the lines being prolonged, would meet in the centre.

A right hand oblique arch is that, when, by approaching one of its elevations, the right hand abutment advances before the left.

A left hand oblique arch is that, when by approaching one of its elevations, the left hand abutment advances before the right.

If the segment of a cylinder be cut, by two parallel planes, obliquely to the axis, and if the cylindric surface between the planes be developed, the curves which are developments of the surface and the sections, will be identical, and will each be bisected by the straight line which joins each extremity, and the two straight lines will be parallel.

The projections of spiral lines, in the same cylindric surface which have parallel developments, are identical curve lines.

A cylindric spiral is a line of double curvature, and has all its parts equally inflected.

The developement of a cylindric spiral is a straight line.

A straight line drawn through any given point on the surface of a cylinder, will be parallel to the axis.

A circle drawn through any given point in the surface of a cylinder, will be in a plane perpendicular to the axis.

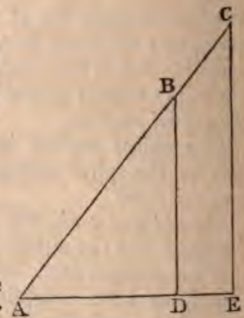
The developement of the arc of a circle is a straight line, and is the length of that arc.

Therefore if on the surface of a cylinder be given a spiral line, and if there be drawn on that surface a circle through one extremity of the line, and a straight line through the other; in the developement of the cylindric surface, the three lines shall form a right-angled triangle of which the developement of the spiral shall be the hypothenuse, the length of the circular arc which may be called the base, one of the sides which contain the right angle, and the other side may be called the atlitude of the spiral.

The angle of inclination of a spiral is the angle which the developement of the spiral makes with its base or with the line which is the length of the circular arc.

The lenghts of two spirals having equal inclinations are to one another as their bases or as their altitudes.

Let $A\ C$ be the developement of a spiral, and let $C\ E$ be parallel, and $A\ E$ perpendicular to the axis of the cylinder. In $A\ C$ take any point B, and draw $B\ D$ parallel to $C\ E$, then the triangles $A\ E\ C$, $A\ D\ B$ are similar, and therefore have equal inclinations; hence



$$AB : AD :: AC : AE$$

$$\text{or} \quad AB : AC :: AD : AE$$

Again

$$AB : BD :: AC : CE$$

$$\text{or} \quad AB : AC :: BD : CE$$

Hence the lengths of two spirals having the same inclination are to one another as their bases or altitudes.

If the bases and altitudes of two spiral lines have the same proportion, the two spirals have the same angle of inclination, and may form one continued spiral.

If the developements of two spiral lines having equal altitudes, but unequal bases be bent upon two concentric cylindric surfaces, the least upon the concave and the greater upon the convex surface, so as to have their proper altitudes, the two spiral lines shall be in a spiral surface, or will form the bed lines of a course of stones, ; moreover the difference of the angles of inclination of the bed lines is equal to the angle of the twist.

PROBLEM I.

To find the development and plan of the intrados of an oblique arch with spiral joints; given the width of the aperture, the height of the intrados, the angle of obliquity, the length of one of the springing lines, and the number of arch stones in each elevation.

The method of the developement of the curve of the oblique section of a cylinder has already been shown (Prob. xv., page 18); but, for the sake of connection, the entire operation is here shown.

Draw *A C*, and make *A C* equal to the width of the aperture.—Upon *A C*, as a chord with the height of the intrados, describe the arc *A B C*. Prolong *C A* to *D*, and make *A D* equal to the length of the arc *A B C*. Draw *A G* perpendicular to *C D*, and make *A G* equal to the length of the springing line. Parallel to *A G*, through *D* draw *F E*, and through *C* draw *f H*. Make *D E* and *C H* each equal to the distance of obliquity. Join *A E* and *A H*, and complete the parallograms *A H f G*, *A E F G*. *A H f G* is the outline of the plan, the lengths of the faces being *A H*, *G f*; and the lengths of the abutments or springing lines *A G*, *H f*. The parallogram *A E F G* is the outline of the developement of the concave surface; except that instead of the straight lines *A E*, *G F* there are also curves *A E*, *G F* which are the developements of those portions of the intrados adjacent to each face of the arch.

Divide the arc *A B C* at the points 1, 2, 3, &c., and the straight line *A D* at the points 1, 2, 3, &c., each into an equal number of equal parts. From the points of division 1, 2, 3, &c., in the arc *A B C*, and from the points 1, 2, 3, &c., in the straight line *A D*, draw 1 *α n*, 2 *α n*, 3 *α n*, &c.; 1 *n*, 2 *n*, 3 *n*, &c.; and let 1 *α n*, 2 *α n*, 3 *α n*, &c., meet the straight lIne *A H* in the points *n*, *n*, *n*, &c., and intersect the straight line *A C* in *α*, *α*, *α*, &c. Make the distances 1 *n*, 2 *n*, 3 *n*, &c., respectively, equal to *α n*, *α n*, *α n*, &c. From *A* through the points *n*, *n*, *n*, &c., draw the curve line *A n n n*...*E*. With the edge of a thin slip of wood formed to the curve *A E*, draw the opposite identical curve *G F*, and the curves *A E*, and *G F* are each bisected by the straight lines *A E*, *G F*, one half of each curve being within, and the other half without the bisecting line, which we shall here call the regulating line.

Divide the straight line *A E*, which is the length of the face of the arch, into as many equal parts as there are arch stones, which in the present example are seven, being designed for a small culvert.—Through the points of division draw lines perpendicular to *A E*, and let one of these lines *k F*, meet the springing line *E F*, in the extremity *F*. Now, if lines were drawn through the points of division in *E k* parallel to *k F*, the springing line *E F* would be divided into as many equal parts as the number of equal parts contained in *E k*, which is here six out of the seven, into which the whole line *A E* is divided.

As there are two methods of drawing the line *k L*, we shall suppose it drawn as here done, reserving the discussion of the mode which may be adopted for the conclusion.

Divide *A G* into six equal parts, at the points *N, P, R, T, V* and draw *N O, P Q, R S, T U, V W,* parallel to *A E,* and *E F* will also be divided into six equal parts at the points *O, Q, S, U, W,* which gives the lengths *E O, O Q, Q S,* &c., of the springers. From, or through the points of division in *A E,* and from the springing points *O, Q. S,* &c., *N, P, R,* &c., in *E F, A G,* draw lines parallel to *k F,* so as to meet the curve lines *A E, G F,* viz. *A E* in the points *t, t, t,* &c., and *G F* in the points *z, z, z,* &c. The parallel lines *t z, t z, t z,* &c , are the developments of the bed lines, and the parallel lines *N O, P G, R S,* &c , are the developements of the spirals in which each series of joint lines are parts. Perpendicular to *A G* draw *t t, t t, t t,* &c., meeting the straight line *A H* in the points *t, t, t,* &c., and the points *t, t, t,* &c., in the straight line *A H* are the ends of the projections of the bed line spirals.

Prolong *A L* the developement of one of the bed line spirals and the springing line *E F* to meet each other in *J.* Then, in order to construct the plan of the spirals of the bed and joint lines, it will be sufficient to show the projection of one of each kind. Through the points 1, 2, 3, &c., in the arc *A B C* parallel to *A G* draw the straight lines *α a b, α a b, α a b,* &c. Let *a, a, a,* &c., be any number of points in *A L,* or in *A L J.* Perpendicular to *A G* draw *a a, a a, a a,* &c., meeting *α a b, α a b, α a b,* &c., in the points *a, a, a,* &c. From *A* through the points *a, a, a,* &c., draw the curve line *A a a a ... l j* which is the projection of one of the bed line spirals corresponding to the developement *A a a a ... L J* of the same spiral. As the projection of spiral lines which have parallel developments are identical curve lines, and since we have the springing points *N, P, R, T, V* as also the ends *t, t,* &c., in *A H,* the remaining projections may be drawn as before shown by the edge of a thin slip of wood formed to the curve *A a a a ... l j* observing that each end of the edge thus formed must be placed upon the springing points in each springing line, or at least one of the ends upon one of the springing lines and the edge successively upon the points *t, t, t,* &c. In the same manner may be found the curve line *V b b b ... w* which is the projection of the same joint line of which the straight line *V b b b ... W* is the developement. The remaining curves of the joint lines may be drawn as observed in the curves of the bed lines.

PROBLEM II.

Given the same things as in the preceeding problem, to find the elevation of one of the faces on a plane parallel to that face.

In drawing the plan or elevation of an oblique arch, the projections of all spirals which have parallel developements are identical curve lines whether in a plan which is made on a plane parallel to the axis or in an elevation which is made on a plane inclined to the axis; the projections on a plan differ considerably from those in an elevation.

In drawing the elevations of oblique arches we must first find the projections of lines parallel to the axis of the cylinder passing through the points of division in the arc, then the projections of the spirals will be found by drawing straight lines, from the points in the projection of a spiral from the plan, to meet the lines which are parallel to the axis perpendicularly in the corresponding points of the projection of the spiral to be found, then drawing the curves through the points in the succeeding parallel lines.

Find the plan and developement as explained in the preceding problem.

Draw the straight lines $p\,q$, $r\,s$, $t\,u$, $v\,w$, $x\,z$, &c., parallel to $A\,H$, making the distance between $p\,q$, $r\,s$, equal to the distance of the first joint below the springing line or about 6 inches, which is the usual allowance; also make the distances of $t\,u$, $v\,w$, $x\,z$, &c., above the springing line $r\,s$ respestively equal to $n\,1$, $n\,2$, $n\,3$, *&c.* the heights of the ordinates of the arc $G\,K\,L$ above the chord $G\,L$. The lines $p\,q$, $r\,s$, $t\,u$, &c, are those which are parallel to the axis. Perpendicular to $A\,H$ draw $i\,i$, $j\,j$, $k\,k$, &c., meeting $r\,s$ in the points i, j, k, &c., and i, j, k, are the projections, of the bottoms of the indentations of the springers; also perpendicular to $A\,H$ draw $l\,l$, $m\,m$, $n\,n$, &c., meeting $t\,u$, $v\,w$, $x\,z$, &c., respectively at the points l, m, n, &c., and draw the curve line $i\,l\,m\,n$ ... which is the projection of one of the courses of spiral joint lines. In the same manner the entire elevation may be completed, as is evident from the figure.

In the projections of the spiral lines in the elevation, the plane of projection is not parallel to the axis of the cylindric surface as is the case with the plane of projection upon which the plan is made. The projections of the spiral lines upon the plan are regular curves, but those in the elevation are not so uniform.

PROBLEM III.

The same things being given to construct the longitudinal section.

This will be done in a similar manner to what has just been shown in the last problem, as will be evident by inspecting plate 19, and therefore any particular explanation will be unnecessary, observing that the curve line $r\,s\,t\,u$ No. 3 is the projection of the curve line $r\,s\,t\,u$ No. 1, and is therefore the spiral line of which $r\,s\,t\,u$ No. 2 is the plan.

PROBLEM IV.

Having the length of the arc of the right section of the convex surface of an oblique arch, and the developement of the concave surface to find the developement of the convex surface.

Let the semicircle $a' b' c'$ be the right section of the convex surface, $A B C$ being that of the concave surface, and let $I E F G$ be the developement of the concave surface constructed as in Problem I, $I G$ and $E F$ being the springing lines. The number of springers in this example are five; therefore, $I G$ is divided into five equal parts at the points N, P, R, T, and $E F$ is also divided into five equal parts, at the points O, Q, S, U. The parallel lines $N O, P Q, R S, T U$, which are the developements of the joint line spirals, are each divided by the developements of the bed line spirals into as many equal parts as there are arch stones in each elevation, viz., nine, as in this example. Therefore, divide the parallel lines $N O, P Q, R S, T U$, each into nine equal parts.

Prolong $A D$ to d. In $D d$ the part prolonged make $a d$ equal to the length of the arc $a' b' c'$, and form the outline $a j x z$, in the same manner as $I E F G$ the developement of the concave surface, $a z, j x$, being the springing lines of this development. Parallel to $A D$ draw $N n, P p, R r, T t$, meeting $a z$ in n, p, r, t, as also parallel to $A d$ draw $O o, Q q, S s, U u$, meeting $j x$ in the points o, q, s, u.—Join $n o, p q, r s, t u$, and the lines $n o, p q, r s, t u$ are the developements of the joint lines in the convex surface, and are parallel to each other, and are also equidistant. Divide the remote parallels $n o$, $t u$, each into nine equal parts; and as the lines $n o, t u$, comprise three steps or springers, the corresponding points through which the bed lines pass will be three parts in advance from t, further than from n. Therefore, the bed line developement $n y$, drawn from n in $n o$, will pass through the point 3, in $t u$, and will terminate in the curve $z x$ at y. Through each of the eight points of division in $n o$ draw lines parallel to $n y$ to meet the curve $z x$ and the curve $a j$. From the springing points p, r, t, draw lines parallel to $n y$ to meet the curve $z x$, and from the springing point o draw a line parallel to $n y$ to meet the curve $a j$, and we shall have the whole developement of the convex surface as comprised by the figure $a j x z$.

In plate 21, the two developements are represented as if they were one diagram comprised by the figure $a j x z$, inorder to show that the angle of the twist is nothing more than the angle made by the developements of two bed line spirals, viz., one in the concave and the other in the convex surface.

In the right angled triangle $W X Y$, draw $X Y$ parallel to the chord or diameter $A C$, $X W$ perpendicular to $X Y$. Make $X Y$ equal to the radius $A I$ of the concave surface, and draw $Y W$ parallel to $D H$, which is the developement of one of the bed line spirals. Prolong $X Y$ to Z, and make $Y Z$ equal to $A a'$. Join $Z W$, and $Z W$ shall be parallel to $J H$, the developement of one of the bed lines of the convex surface; therefore, the angle $Y W Z$ is equal to the angle $D H J$.

PROBLEM V.

Given the chord and height of the intrados, the distance of obliquity, the distance between the two faces, the breadth of the beds, and the number of arch-stones, to construct the templets for working the arch-stones.

Draw *A C* equal to the chord, and with the height of the intrados describe the arc *A B C*; prolong *C A* to *D*, and make *A D* equal to the length of the arc. Draw *C H* and *D E* perpendicular to *C D*, and make *C H* and *D E* equal to the distance of obliquity. Join *A H*, *A E*. Describe the curve *A E* in the same manner as directed in Problem XV, page xviii Introduction. Prolong *E D* to F, and *H C* to *Q'*, and draw *A G* parallel to *E F*. Perpendicular to *A H* draw *A u*, and make *A u* equal to the distance between the faces. Through *u* draw G *Q'* parallel to *A H*, and draw *G F* parallel to *A E*. Draw the curve line *G F*, and *A H Q' G* is the outline of the plan, *A E F G* the outline of the developement. In order to prevent two joints from meeting each other it is necessary that the number of arch stones in each face should be an odd number; we shall therefore suppose this number to be nine. Draw *F K* meeting the straight line *A E* perpendicularly in *K*. Divide *E K* into five equal parts, and *K A* into four equal parts, so that the whole line *E A* may consist of nine parts as nearly equal to one another as the case will admit of. Now there will be as many springers in *E F* as in the part *E K*, and as many in *A G* as in *E F*; therefore, divide *E F* into five equal parts at the points *N*, *P*, *R*, *T*. Draw *N O*, *P Q*, *R S*, *T U*, parallel to *E A*, meeting *A G* in *O*, *Q*, *S*, *U*. From the points *N*, *P*, *R*, *T*, and *O*, *Q*, *S*, *U*, as also from the points of division in *A K*, draw lines parallel to *K F* to meet the curve lines *A E*, *G F*. Then *K F*, and all the lines parallel to *K F* are the developements of the bed lines, and the alternate portions of the parallel lines *N O*, *P Q*, *R S*, &c., are the developements of the joint lines. Draw *a e* parallel to *A C*, of any convenient length, and with the radius *I A* or *I C* describe the arc *a c e*; draw *a f* parallel to *A G*, and *e f* parallel to *K F*. Draw *e m* parallel to *A G*, and *f m* parallel to *E A*. In *a e* take any convenient number of points 1, 2, 3, &c. Draw the straight lines 1-1, 2-2, 3-3, &c., parallel to *A G* intersecting the arc *a c e* in the points *b*, *c*, *d*, *&c.*, and *f e* in the points 1, 2, 3, &c., and meeting *f m* in the points 1, 2, 3, &c. Perpendicular to *f e* draw 1 *g*, 2 *h*, 3 *i*, &c., and perpendicular to *f m* draw 1 *j*, 2 *k*, 3 *l*, &c. Make 1 *g*, 2 *h*, 3 *i*, &c., respectively equal to 1 *b*, 2 *c*, 3 *d*, &c. and likewise make 1 *j*, 2 *k*, 3 *l*, *&c.*, respectively equal to 1 *b*, 2 *c*, 3 *d*, &c. Draw the curves *f g h i e*, *f j k l m*. Make two templets No. 1, No. 2 to form the segments *f h e*, *f k m*. Make also two arch squares No 3, No 4, having the inner edges of the one limb curved and the other inner edge straight, the straight edge being perpendicular to the under edge of the curved limb, observing that the curved limbs, *q r s*, *n o p*, must be identical to the half segments of No. 2, No. 1, viz. *q r s* the same as *h l m*, and *n o p* the same as *h i e*.

In the right-angled triangle $W X Y$ draw $X W$ parallel to $A G$, and $X Y$ parallel to $D A$. Make $X Y$ equal to the radius $I A$ or $I C$, and draw $Y W$ parallel to $K F$. Prolong $X Y$ to Z, and make $Y Z$ equal to the breadth of the beds. Join $Z W$, and the angle $Y W Z$ is the angle of the twist.

TO WORK ONE OF THE ARCH STONES.

Besides the templets already shown, prepare two straight edges, the one parallel, and the other broader at one end containing the angle of the twist. We shall suppose that all the faces of the stone are rectangler and that the stone is laid upon one of its beds, and consequently the other bed uppermost, and that you are placed adjacent to the soffit of the stone. Apply the templet No. 1, so that the curved edge $f\ g\ h\ i\ e$ may rest upon each end of the arris next to you, then upon the surface of the stone draw a line by the curved edge, and the line thus drawn will be the bed line between the soffit and that bed, observing that the curve must be convex to the opposite arris of the stone or concave to the adjacent arris. Apply the parallel strait edge upon the arris next to you which is to be cut away, and the other straight edge which contains the angle of the twist, so that the planes of two faces of these straight edges may be parallel to each other, and at such a distance as to be equal to the intended breadth of the beds; then the straight edge which contains the angle of the twist must be sunk into the stone until the upper edges of the two straight edges are out of winding or are in one plane. The bottom of the chisel draught will then make an angle with the surface of the stone equal to the angle of the twist. If the arch is right handed, the broader end of the straight edge must be applied next to the left hand of the stone; but if the arch is left handed, the broader end must be applied next the right hand end of the stone The protuberant part between the bottom of the chisel draught, and the bed line must be taken away until the surface agrees every where with a straight edge applied from any point perpendicular to the bed line or to the curve which forms the bed line. The spiral surface of the one bed being thus formed, we now proceed to form the soffit. Apply the arch square No. 3 in the same manner as a common square, each limb being perpendcular to the arris or to a tangent to the curve which forms the arris or bed line, so that the straight edge $q\ v$ may coincide with the surface of the bed, and the curved edge $q\ r\ s$ may rest upon the stone which is attached to the soffit, and which is to be wrought awa y. Repeat the application until the curved edge $q\ r\ s$ coincide with the surface of the soffit which will then be that of a cylinder. Having formed the cylindric surface, guage the soffit to its breadth, and proceed with the same arch square to work the other bed, it need hardly be observed that the curved edge must be applied to the soffit or cylindric surface, and consequently the straight edge upon the winding surface of the bed.

ON THE

HISTORY AND THEORY

OF

OBLIQUE BRIDGES.

THE rapid increase of the commerce and manufactures of this country having of late years demanded increased facilities of communication, the demand has been met in the first instance by the construction of canals, and more recently by the introduction of railways, and the application of steam power to the purposes of locomotion.

But as it is always the result of human inventions to make further demands upon human ingenuity, the formation of canals and railways has opened a field previously unknown to the genius of our engineers, and afforded them splendid opportunities for the exercise of their talents in overcoming the obstacles which are presented by nature to the execution of these great public undertakings.

The graceful undulations of the country, while they constitute the greatest charm of the landscape, and offer the most delightful picture to the lover of nature, present the most serious difficulties to the operations of art, and tax to the uttermost the professional skill of the architect and engineer, to preserve the level line which the construction of these works imperiously requires.

It is to these difficulties, therefore, that we owe the stupendous monuments of human intellect which are scattered over the face of the country; the tunnels, bridges, aqueducts, and embankments which excite our admiration at the genius by which they were projected, and the industry by which they were carried into execution, and thus we are indebted to the very cir-

cumstances which threaten to render works of this description totally impracticable, for the objects which lend these works their chief attraction to the scientific mind.

There is perhaps none of these contrivances in which the application of science to the common affairs of life is better exemplified than in that which forms the subject of this treatise.

The oblique arch is an invention of comparatively recent date, and until the general adoption of railways, and the developement of the powers of the locomotive engine, was comparatively of little consequence, but the immense speed of travelling attained by these means rendering it urgent to preserve the most direct line, and thus precluding to a great extent the employment of bridges on the common plan, has caused the general adoption of oblique bridges on all the lines of railway now in progress and made it a matter of importance that the theory of their construction should be fully understood.

The late eminent engineer, William Chapman, a native of Newcastle on Tyne, seems entitled to the honour of originating the idea of oblique bridges in their present form; but although fully aware of the difficulty of constructing them, he does not appear to have laid down any sound principle by which it might be done.

In an article in *Rees' Cyclopedia*, title "Oblique Arch," which appears to be the first notice on record of the employment of the oblique arch, properly so called, he explains the substitutes for oblique bridges then in use, and the circumstances which led to their introduction, as follows:—

"Wherever high roads run oblique to the course of any river, rivulet, drain or canal, necessary to be passed over by a bridge, the direction of the *former* is generally varied so as to be rectangular to the course of the *latter* unless in small streams, over which, when their course is not made to suit the road, there are several instances of the construction of what are termed *Skew Bridges*.

"These, with the exceptions which will be afterwards mentioned, have been built in the usual manner of laying each course of stones or bricks of the arch parallel to the line of the abutment and beveling off their ends on each exterior face of the arch in a line correspondent with the intended direction of the road over the bridge."

After pointing out the insecurity of this mode of building oblique bridges, he proceeds—

"These circumstances have prevented cautious builders from adopting this method, and induced them, in a few instances, to

build the arch square to its abutments and run the parapets oblique to coincide with the line of road; leaving alternate triangles of the arch on the outside which has a disagreeable appearance, and has seldom been used; therefore, in general, unless the courses of streams and canals were made rectangular to the road, the line of the latter has been altered so as to admit of a direct passage over the water, which upon high roads when not curved to a considerable distance, is inconvenient if not dangerous; and particularly so to travellers in the night time; from which cause the skew bridges described were with all their imperfections, occasionally had recourse to; and the writer of this article has never heard of any alteration in the form prior to the year 1787. At this time he had the direction of the County of Kildare Canal, a branch from the Grand Canal of Ireland, to the town of Naas."

"In the course of conducting the work, several of the directors of that canal were anxious to have the line of the roads unvaried; therefore our author was led to consider whether the usual imperfect method could not be set aside by the substitution of one on sound principles; and it then occurred that *its leading feature must be*, that *the joints of the voissoirs* whether of brick or stone, *should be rectangular with the face of the oblique arch* in place of parallel with its abutments, and consequently the bridges over the county of Kildare canal were made to suit the line of road, although the obliquity of one of them was carried to an extent beyond what he deemed eligible in practice, as will appear from the observations made on Finlay Bridge, near the town of Naas, which deviated 51° from a rectangle with the canal, and consequently formed the acute angle of 39° with its abutments. Its span in that direction was 25 feet, and its pitch 5 feet 6 inches, or nearly 2-9ths of its span."

After further discussing the subject, Mr. Chapman comes to the following conclusion respecting the construction of oblique arches on an improved plan:—

"*The impost course must be serrated,*" "*and as the lines in which the beds of the voissoirs run are obviously spiral lines, it follows that the soffit of each stone must be curved in that direction, and likewise it must be twisted in its sommering, which although not insuperable difficulties, are so in such a degree as combined with the indented form of the impost, render it advisable to use brick both for the impost and arch, or at most to be contented with the use of stone only for the quoins and their necessary imposts*, in the forming of which intelligent stone cutters will be requisite." "It is apparent that the head of each

voissoir on that side of the arch where its face forms an acute angle with the abutment must make an obtuse angle with its soffit decreasing in their approach to the crown of the arch, and thence forward becoming acute and increasing as they advance to the other impost where the face of the arch forms an obtuse angle with its abutment; therefore the different sides of the same voissoir must form different angles of elevation or depression from the rectangle with its head."

"*A Geometrical mode of forming each voissoir would be complicated.*"

Mr. Chapman seems to think that the construction of oblique arches upon geometrical principles would be difficult, and having broached what the leading features of their construction ought to be, leaves the execution of the work to the ingenuity of the mason, without laying down any principles to guide him.

Mr. Tredgold, in the *Encyclopedia Britannica*, has also written upon the oblique arch. In his construction, the beds of the courses of arch stones meet each other in planes perpendicular to the face of the arch, and to the intrados only in the curve where it intersects that face, and as the beds advance to the opposite side face, their planes cut the intrados the more obliquely. Mr. Tredgold's talents are well known to the public; I shall, however, leave the reader to judge of the eligibility of this method.

These were all the principles which were in the possession of the mason till the publication of the *Treatise on Masonry and Stone-cutting* by me in 1828, the theory of oblique arches contained in which, will be more fully discussed hereafter.

The next theory is that propounded by Mr. Fox, in the *London and Edinburgh Philosophical Magazine*, published April, 1836 (eight years after my *Treatise on Masonry*), who, after remarking on the advantages of skew bridges, or oblique arches over these of the common construction, introduces his theory as follows:—

"As many practical men with whom I am acquainted have experienced considerable difficulty in the construction of skew bridges, I was led to turn my attention to the subject; and have at length succeeded in rendering the principles of it easy to be understood."

Now it certainly would appear that Mr. Fox is either chargeable with ignorance of the subject on which he treats, or with want of that candour which induces men of science to allow their predecessors the honour and credit of what they have written on the same subject. For the impression produced upon the mind of any one reading the introduction to his paper

would be, that Mr. Fox was the first who had "turned his attention to the subject," and "succeeded in rendering the principles of it easy to be understood." I could, however, have been very content to pass Mr. Fox's paper, with an exposition of his principles, trusting that to every person conversant with the subject, my treatise had already spoken for itself; but that that gentleman, in a subsequent paper in the same magazine (March, 1837, page 167), after being informed that a rule for the construction of the oblique arch was in existence some years prior to the publication of his theory, avoiding altogether the question of his previous knowledge of such a rule, enters into a discussion of the question "whether his rule is identical with mine, and, if not, which is the better of the two."

This question he settles much to his own satisfaction, and pronounces his judgement in the following logical sentence:—

"I trust that what has been brought forward shows that Mr. Nicholson's rule and mine are *not alike*, and *therefore*, *that he has no "prior claim to mine, and that my rule possesses advantages not to be found in his*."

In the *Philosophical Magazine*, before referred to, April, 1836, page 300, Mr. Fox says,

"The principle which I have adopted is, to work the stones in the form of a spiral quadrilateral solid, wrapped round a cylinder, or, in plainer language, the principle of a square-threaded screw; hence, it becomes quite evident, that the transverse sections of all these spiral stones are the same throughout the whole arch. It will be obvious, that the beds of the stones should be worked into true spiral planes; but I am not aware that any rule has yet been published that would enable the stones to be wrought at the quarry into the desired form, or of any rule by which the true angle at which the courses cross the axis of the bridge is determined."

I shall just notice, that the expressions "to work the stones in the form of a spiral quadrilateral solid wrapped round a cylinder," and "that the beds of the stones should be worked into true spiral planes," convey no idea of geometrical principle: a thin piece of paper may naturally be bent round a cylinder, but such a solid can only be constructed to fit the curved surface, and a spiral surface cannot be called a spiral plane; there are many kinds of surfaces, but only one that is called a plane, or plane surface.

With respect to the concluding paragraph of Mr. Fox's remarks, I may state that I am in possession of a letter from Mr. James Hogg, to Henry Welch, Esq., Bridge-surveyor to

cumstances which threaten to render works of this description totally impracticable, for the objects which lend these works their chief attraction to the scientific mind.

There is perhaps none of these contrivances in which the application of science to the common affairs of life is better exemplified than in that which forms the subject of this treatise.

The oblique arch is an invention of comparatively recent date, and until the general adoption of railways, and the developement of the powers of the locomotive engine, was comparatively of little consequence, but the immense speed of travelling attained by these means rendering it urgent to preserve the most direct line, and thus precluding to a great extent the employment of bridges on the common plan, has caused the general adoption of oblique bridges on all the lines of railway now in progress and made it a matter of importance that the theory of their construction should be fully understood.

The late eminent engineer, William Chapman, a native of Newcastle on Tyne, seems entitled to the honour of originating the idea of oblique bridges in their present form; but although fully aware of the difficulty of constructing them, he does not appear to have laid down any sound principle by which it might be done.

In an article in *Rees' Cyclopedia*, title "Oblique Arch," which appears to be the first notice on record of the employment of the oblique arch, properly so called, he explains the substitutes for oblique bridges then in use, and the circumstances which led to their introduction, as follows:—

"Wherever high roads run oblique to the course of any river, rivulet, drain or canal, necessary to be passed over by a bridge, the direction of the *former* is generally varied so as to be rectangular to the course of the *latter* unless in small streams, over which, when their course is not made to suit the road, there are several instances of the construction of what are termed *Skew Bridges*.

"These, with the exceptions which will be afterwards mentioned, have been built in the usual manner of laying each course of stones or bricks of the arch parallel to the line of the abutment and beveling off their ends on each exterior face of the arch in a line correspondent with the intended direction of the road over the bridge."

After pointing out the insecurity of this mode of building oblique bridges, he proceeds—

"These circumstances have prevented cautious builders from adopting this method, and induced them, in a few instances, to

"On further consi-deration I discovered a more eligible mode of laying out the lines."

"To get the thrust strictly correct, I have supposed the arch to be cut into two rings of equal thickness." "And having considered the external ring as removed, have proceeded to develope the outside surface of the remaining one; this I shall here afterwards speak of as the intermediate development, as it is the development of a surface midway between the extrados and the soffit or intrados."

"Upon this intermediate development I place the approximate line, and then draw all the courses square to it."

"Having explained the mode of setting out the beds of the stones, a little may now be said on the situation of the cross joints; byhese will be understood the joints between the various stones constituting a complete course."

"My first idea was to commence by working the soffit, and this was the mode employed."

"Having obtained an elastic mould cut to the angle at which the joints of the soffit cross the axis of the bridge, the workman by means of this gets an oblique line on that surface of the stone which he intends for the soffit." "This oblique line thus obtained will be parallel to the axis of the bridge. The workman then proceeds to chisel out a groove (or what is by masons called the chisel draught) along this line, of sufficient depth for what he knows will be required for the hollowing of the stone."

"He then takes two wooden moulds," "which are portions of the same circle as the soffit itself. A mark being placed upon the centre of each of these moulds, the workman then proceeds to sink them into the stones at right angles to the chisel draught;" "and, in such a manner, that the centre marks shall be in the chisel draught, and the upper edges of the moulds, which are straight, shall be in the same plane, or what is commonly called out of winding. It will now be obvious that these last two grooves will form true portions of the soffit itself, and, therefore, that the workman has nothing to do but to work out the remainder of the stone with a straight edge always kept parallel with the first draught, and sunk to the bottom of the two draughts which were worked by the curved moulds. Having obtained the hollowed surface, an elastic mould of the exact size of the soffit of each stone is pressed into it, by which the stone being marked, we obtain all the lines of the soffit itself."

"It will now be quite evident that the beds may be obtained by making use of a square, one limb of which shall be made to the curvature of the soffit, and the other the radius of this curve;

always taking care that this square is kept at right-angles to the axis."

All this, so far, is in effect the same as the rules given by me in my *Treatise on Masonry.*

"The first few stones were wrought in this manner; but, finding it very difficult to prevent the workman from getting his soffit a little on one side, by which means he wasted much of the stone on one bed, and rendered the other deficient,* I had recourse to a method which I shall describe. Having provided two straight edges, the one parallel, and the other containing the angle of the twist, we proceeded to work one of the beds by chiselling two draughts along the stone, so that these straight edges being kept at a proper distance from each other, were let into the stone until they were out of winding on their upper edges."

"Having finished one bed by straight edges, we then obtained the soffits and other beds by means of the square before mentioned."

Thus, in forming the arch stones, Mr. Fox gives two methods: the first is to work the intrados, or soffit, by the same templets, and then the beds to the twist by the same instrument, as previously shown in my *Treatise on Stone-cutting.*

In the second method, which he prefers, he reverses the process by working one bed to the twist by two straight edges, and then the soffit is formed to the curved surface of the cylinder, by the same imperfect instrument which was used in the first case for working the beds. The principle of the first method is correct; the second method supposes that the beds can be wrought with a straight edge to the angle of the twist, independent of previously forming the cylindric surface of the intrados; but this supposition is not correct; for the only direction in which a straight line passing through any given point in the spiral surface of the bed is in a straight line perpendicular to the axis of the cylinder; but if the spiral surface be cut by a plane perpendicular to a straight line passing through any point in that surface perpendicularly to the axis, the section will be very nearly in a straight line, and, practically considered, the spiral bed may be formed to the twist by straight lines nearly perpendicular to each other: in this respect, how-

* The reason of this uncertainty is in consequence of the straight draught not being made to pass through the centre of the face of the stone intended for the soffit; if this is not the case, the soffit will be wrought more upon one side than the other, and therefore to work one of the beds, so much stone will often be wasted as to render the other deficient, if the stone is not sufficiently thick.

ON THE

HISTORY AND THEORY

OF

OBLIQUE BRIDGES.

The rapid increase of the commerce and manufactures of this country having of late years demanded increased facilities of communication, the demand has been met in the first instance by the construction of canals, and more recently by the introduction of railways, and the application of steam power to the purposes of locomotion.

But as it is always the result of human inventions to make further demands upon human ingenuity, the formation of canals and railways has opened a field previously unknown to the genius of our engineers, and afforded them splendid opportunities for the exercise of their talents in overcoming the obstacles which are presented by nature to the execution of these great public undertakings.

The graceful undulations of the country, while they constitute the greatest charm of the landscape, and offer the most delightful picture to the lover of nature, present the most serious difficulties to the operations of art, and tax to the uttermost the professional skill of the architect and engineer, to preserve the level line which the construction of these works imperiously requires.

It is to these difficulties, therefore, that we owe the stupendous monuments of human intellect which are scattered over the face of the country; the tunnels, bridges, aqueducts, and embankments which excite our admiration at the genius by which they were projected, and the industry by which they were carried into execution, and thus we are indebted to the very cir-

cumstances which threaten to render works of this description totally impracticable, for the objects which lend these works their chief attraction to the scientific mind.

There is perhaps none of these contrivances in which the application of science to the common affairs of life is better exemplified than in that which forms the subject of this treatise.

The oblique arch is an invention of comparatively recent date, and until the general adoption of railways, and the developement of the powers of the locomotive engine, was comparatively of little consequence, but the immense speed of travelling attained by these means rendering it urgent to preserve the most direct line, and thus precluding to a great extent the employment of bridges on the common plan, has caused the general adoption of oblique bridges on all the lines of railway now in progress and made it a matter of importance that the theory of their construction should be fully understood.

The late eminent engineer, William Chapman, a native of Newcastle on Tyne, seems entitled to the honour of originating the idea of oblique bridges in their present form; but although fully aware of the difficulty of constructing them, he does not appear to have laid down any sound principle by which it might be done.

In an article in *Rees' Cyclopedia*, title "Oblique Arch," which appears to be the first notice on record of the employment of the oblique arch, properly so called, he explains the substitutes for oblique bridges then in use, and the circumstances which led to their introduction, as follows:—

"Wherever high roads run oblique to the course of any river, rivulet, drain or canal, necessary to be passed over by a bridge, the direction of the *former* is generally varied so as to be rectangular to the course of the *latter* unless in small streams, over which, when their course is not made to suit the road, there are several instances of the construction of what are termed *Skew Bridges*.

"These, with the exceptions which will be afterwards mentioned, have been built in the usual manner of laying each course of stones or bricks of the arch parallel to the line of the abutment and beveling off their ends on each exterior face of the arch in a line correspondent with the intended direction of the road over the bridge."

After pointing out the insecurity of this mode of building oblique bridges, he proceeds—

"These circumstances have prevented cautious builders from adopting this method, and induced them, in a few instances, to

build the arch square to its abutments and run the parapets oblique to coincide with the line of road; leaving alternate triangles of the arch on the outside which has a disagreeable appearance, and has seldom been used; therefore, in general, unless the courses of streams and canals were made rectangular to the road, the line of the latter has been altered so as to admit of a direct passage over the water, which upon high roads when not curved to a considerable distance, is inconvenient if not dangerous; and particularly so to travellers in the night time; from which cause the skew bridges described were with all their imperfections, occasionally had recourse to; and the writer of this article has never heard of any alteration in the form prior to the year 1787. At this time he had the direction of the County of Kildare Canal, a branch from the Grand Canal of Ireland, to the town of Naas."

"In the course of conducting the work, several of the directors of that canal were anxious to have the line of the roads unvaried; therefore our author was led to consider whether the usual imperfect method could not be set aside by the substitution of one on sound principles; and it then occurred that *its leading feature must be*, that *the joints of the voissoirs* whether of brick or stone, *should be rectangular with the face of the oblique arch* in place of parallel with its abutments, and consequently the bridges over the county of Kildare canal were made to suit the line of road, although the obliquity of one of them was carried to an extent beyond what he deemed eligible in practice, as will appear from the observations made on Finlay Bridge, near the town of Naas, which deviated 51° from a rectangle with the canal, and consequently formed the acute angle of 39° with its abutments. Its span in that direction was 25 feet, and its pitch 5 feet 6 inches, or nearly 2-9ths of its span."

After further discussing the subject, Mr. Chapman comes to the following conclusion respecting the construction of oblique arches on an improved plan:—

"*The impost course must be serrated,*" "*and as the lines in which the beds of the voissoirs run are obviously spiral lines, it follows that the soffit of each stone must be curved in that direction, and likewise it must be twisted in its sommering, which although not insuperable difficulties, are so in such a degree as combined with the indented form of the impost, render it advisable to use brick both for the impost and arch, or at most to be contented with the use of stone only for the quoins and their necessary imposts*, in the forming of which intelligent stone cutters will be requisite." "It is apparent that the head of each

by my rule they are *necessarily curves* (see fig. 5, of the plate illustrating my paper)."

It may here be remarked, that Mr. Fox has shown no principles from which it may be inferred that the bed-joints in the face are *necessarily curves*. This conclusion has only been drawn by trying the beds in various directions with a straight-edge, in the execution of the work.

It is very true, that in my *Treatise on Stone-cutting*, as well as in the present work, the bed-joints in the face of the arch are all represented by straight lines; for if they had been curves, the curvature would have been so small that the joint lines would not have sensibly varied from straight lines. The true curvature of the joints could not, therefore, have been exhibited in lines, but may be explained in the following manner: —In the bed-joints of the elevation of an oblique arch, the curvature is continually varying upon each side of the arch from the springing, where it is the greatest, to the crown, where it changes into a straight line, and in the elevation the curvature of the bed-line joints are all concave towards the axis minor of the ellipse, which is the section of the cylinder forming the elevation, and the joint-lines are not perpendicular to the curve, but incline nearer to the summit of the arch, in a contrary direction to what plane joints would have done; for, in a semi-cylindric intrados, the bed-joints, in the face of the arch constructed with plane joints, would have radiated to the centre of the semi-ellipse.

What Mr. Fox has said about the oblique arch is a mere outline of the most evident principles, and with regard to explaining difficulties, he has done nothing. He is quite silent as to any method of forming the heading joints, and his method of working the soffit is very liable to error. One of the most difficult things to accomplish in the working of the stones of an oblique arch is, to form those in the face with such accuracy that they shall not want paring after being set; of this difficulty he has not even given a hint that any thing of the kind is required.

The next writer upon the oblique arch is a Mr. Hart. His treatise is dated 1837, and commences thus:—

"In laying the following treatise before the public, I flatter myself that I have thrown some light on a subject hitherto apparently but little understood; my principal aim throughout has been to simplify the construction of oblique arches as much as possible, which I trust will be seen upon an examination of the plates, and their definitions. I have been more anxious to explain them in language suited to the capacities of men engaged in the execution of them, than to embellish the work with scien-

tific terms (which, with many, would require more learning than a knowledge of the work itself), thus rendering it what it professes to be, a useful reference for practical men;—my own experience in the superintendence of work and workmen, having placed before me the necessity of adapting the explanations of drawings to the understandings of those engaged in their execution; for much of the value and beauty of a design depends upon the workmen being acquainted with the principles of the construction upon which they are engaged.

"If, therefore, this treatise assists in conveying the draughtsman's intentions to the workman's understanding, it will answer the intended purpose; for I have no doubt, that the draughtsman will derive some information, as well as the mason and bricklayers, it having been my principal object throughout the arrangement of the plates to make it serviceable to all interested."

"PLATE I *is the drawing of a skew arch, being the segment of a circle on the face, and built with spiral courses according to the principles laid down in the Plate* 3."

It would be useless to follow this author entirely through the description of the first plate, which occupies ten pages: the want of identifying his drawings with the objects which they represent, and the obscure manner in which he treats the subject, makes it difficult to understand his explanations. The parts of an object can only be conceived by inspecting the Plate. It will be sufficient for the purpose to quote some of the most intelligible parts of his explanation, and to show the results to which his ideas lead. After having formed the outline of the development of the intrados, he makes a mould to the undulated curve, which is the development of the circular arc, or the section of the cylindric surface, and the plane of the face of the arch. This curve, from his method of construction, is necessarily divided into equal parts in number equal to that of the arch stones, and by a mould made to this curve he draws the development of the ellipse in the rear elevation; on this he makes the following observations in a note at the bottom of the page.

"By dividing the development on the spiral lines, the stones on the elevation of the arch will be equal, but the soffit course will *not* be parallel. If, on the other hand, it is divided on the straight line $b\,j$, the soffit course will be parallel, but the face of the arch stones will *not* be equal."

The statement here made does not correspond with the diagrams shown on the plate. I should apprehend his meaning to be as follows:—

By dividing the undulated curves into equal parts, the thickness of the stones in the elevation of the arch will be equal; but the thickness of any course of stones in the spiral direction will not be equal. If, on the other hand, the straight lines which join the ends of these curves be divided into equal parts, the thickness of the spiral courses will be equal, but that of the arch stones in the elevation unequal.

"The next thing required is to determine the direction of the bed-joints or spiral courses, and perhaps the best way is to set them out as nearly square from the face of the arch as the construction will admit of, that is as the joint on the spiral line *c n p* will allow; therefore if from the point *b* a line be drawn perpendicular to *b m j* as *b z*, and if *z* does not fall in one of the divisions on the spiral line *c n p*, it must be drawn to the one it is nearest to, preferring, should it come near the middle of a course, to keep it nearest the springing at *c*."

In this as well as in the preceding statement, the undulated curve in the development of the intrados is called a spiral line, which is a term improperly used by Mr. Fox, from whose paper it has been taken.

In an oblique arch, executed according to Hart's principles, cut by a plane parallel to the plane of either of the two elevations, the section would exhibit the joint lines of the stones radiating to the centre of the circular arc which is the section of the cylindric surface, or intrados; but the angles which the joint lines make with the axis continually vary from an obtuse angle on the one side, and from an acute angle on the other, to a right angle at the summit.

The projections of the spiral lines which he exhibits on the plan are not drawn through the opposite angles of rectangles but through the opposite angles of oblique angled parallelograms; two sides of each being parallel to the springing, and the other two parallel to the lines on the plan which are the projections of the two faces of the arc. These lines are the projections of the spiral lines, made by the edges of the beds on the intrados: but the projections of the joint lines made by the ends of the arch stones in any spiral course are all parallel to the lines on the plan which represent the two elevations, and the projections of each spiral course is divided into twice as many equal parts as the number of stones in a course.

Hart, in following out his principle, makes all the sections of the cylinder, which are parallel to the planes of elevation, equal and similar arcs of circles, the sections of the cylinder which are perpendicular to the axis are, therefore, portions of ellipses.

The consequence of this is, that no two stones can be formed alike, or brought to shape by the same moulds. An oblique arch, constructed upon this principle, will not only be deficient in strength, but, from the labour which it would require, would be very expensive.

Mr. G. W. Buck has just published (July, 1839,) his *Essay on the Theory and Practice of Oblique Bridges.* This essay is not explained by geometrical constructions, as hitherto done in practical works, but by trigonometrical formulæ; the necessary angles and lengths are, therefore, principally determined by the aid of calculation. The reader will form some idea of the nature of this publication, from the following extract. He says—

"In carrying this plan out, some mathematical formulæ have necessarily been introduced; they are extremely simple, and the application of them is given in two examples, worked out at length, which must be amply sufficient for all such as are likely to use this little work. It was not designed for the uneducated workman: the subject cannot be reduced to his level and properly treated at the same time. It is written for the use of engineers and architects generally, but principally for those who have the immediate superintendence of public works, and who, being young men, it is to be hoped will not complain of its being too mathematical."—*End of Introduction.*

He concludes his work by the following observations:—

"That many oblique arches are erected which have an elliptical direct section, we are well aware, but consider them deficient in stability, much more difficult to execute, and, consequently, more expensive, especially in masonry; and as far as we have been able to investigate the subject, they do not admit of the application of simple formulæ, similar to those we have established for oblique cylindrical arches. We do not think any combination of circumstances likely to arise in which it would be imperative on the engineer to erect an oblique elliptical arch, and for these reasons have rejected them altogether; nevertheless, we shall feel much gratified if any one would take up the investigation of that subject and convince us of our error, if it be one."

In a note at the bottom of page iii. (Introduction), he says,—"This work was announced for publication about three years back, the author being then urged to do so, but it was afterwards postponed in consequence of finding that his knowledge of the subject was daily increasing by the experience afforded in the construction of a variety of bridges. Since that period the

whole has been remodelled, and the results of subsequent experience and investigation added thereto; and he would have regretted if the work had appeared earlier."

Mr. Buck's *Essay on the Oblique Arch* (as far as we had leisure to examine it), and the present work, have, apparently, been conducted upon the same common principles. Nicholson's Treatise on the Oblique Arch was published in three parts, two of which appeared before Mr. Buck's Essay, part 1 in January, and part 2 in May following, as appears by an advertisement in the *Newcastle Chronicle*, May 4, 1839.

The formula $C\,O=(r+e)$ cot. θ tan. β, is due to Mr. Buck.— It gives the distance below the centre to the point of convergence, into which all the joints in the elevation of the arch meet in the axis minor, supposing that the joints are straight lines, which they are not exactly; having given the angle of obliquity$=\theta$, and the angle in which the bed lines cross the axis of the cylinder$=\beta$, or the angle which a bed line makes with the adjacent springing line. In this formula also r is the radius of the cylinder, $r+e$ the radius of the extrados, e being the breadth of the bed, or thickness of the arch.

He says (page 7) that the expression $C\,O=(r+e)$ cot θ tan. β, included among some others, "are general, that is they are applicable to segments as well as to the semi-circles;" but in page 9 he gives $\dfrac{c \cot^2\theta}{a}(r+e)=C\,O$ "the eccentricity or focal distance below the axis of the cylinder in the oblique segment."

One thing which we consider as defective in Buck's *Essay on Oblique Arches* is, that his intentions are not enunciated under regular heads, so as to call the attention of the reader, he gives no reason for his rules, nor does he show the principles upon which his formulæ depend. The height of the point O figure 7, will depend upon the breadth of the beds.

Mr N. can hardly have meant this as a stricture on Mr Buck's [illegible] formula otherwise perhaps he will explain what he has mistaken e for. It is clear that Mr N. knows nothing about the nature of the distance C O otherwise he would never have given the clumsy and confusing construction at Page 17.

A PRACTICAL METHOD

OF

CONSTRUCTING AS MUCH OF THE DEVELOPEMENT OF

THE OBLIQUE ARCH

TO THE FULL SIZE,

AS WILL BE FOUND SUFFICENT FOR MAKING THE TEMPLETS, FOR THE USE OF THE WORKMAN.

Draw the straight line $O\ C$ (Fig. 1), and in $O\ C$ take any point E. Draw $E\ A$ perpendicular to $O\ C$, and make $E\ B$ equal to the height of the arch. Prolong $B\ E$ to I, and make $B\ I$ equal to the radius of the cylinder. With the radius $I\ B$ describe the arc $B\ C$, and perpendicular to $E\ C$ draw $C\ F$. Make $C\ F$ equal to the distance of obliquity, and join $E\ F$. Make $E\ O$ equal to the length of the arc $B\ C$, and divide the arc $B\ C$, and the straight line $O\ E$ each into the same number of equal parts at the points 1, 2, 3, &c. From the points 1, 2, 3, &c., in the arc $A\ C$ draw $1\ a'$, $2\ b'$, $3\ c'$, &c., meeting $E\ C$ in a', b', c', &c., intersecting $E\ F$ in a, b, c, &c.; and from the points 1, 2, 3, &c., in the straight line $O\ E$ draw $1\ \alpha$, $2\ \beta$, $3\ \gamma$, &c. Make $1\ \alpha$, $2\ \beta$, $3\ \gamma$, &c., respectively equal to $a'\ a$, $b'\ b$, $c'\ c$, &c., and from O through the points α, β, γ, &c., draw the curve $O\ \alpha\ \beta\ \gamma\ A$ which will be half of the developement of the intradosal line. Join $A\ O$, and $A\ O$ is half the line of the subtense.

The construction of figure 1 will become evident by comparing it with figure A, which shows the developement of the intradosal line to the full extent. Figure A is a part of the construction of plate 24, of which see the explanation, page 10. The curve line $O\ \alpha\ \beta\ \gamma\ A$, figure 1 is identical to the curve line $o\ \alpha\ \beta\ \gamma\ A$, figure A, and in figure A the part $o\ \alpha\ \beta\ \gamma\ E$ is identical to the part $o\ \alpha\ \beta\ \gamma\ A$, and both parts make the entire developement of the intradosal line.

Parallel to $E\ C$ in any convenient place, draw the straight line $a\ e$ (Fig. 2) of any convenient length and upon $a\ e$ as a chord with the radius of the cylinder, describe the arc $a\ c\ e$. Draw $a\ f$, $e\ m$ perpendicular to $a\ e$, and draw $e\ f$ perpendicular to $A\ O$. Bisect $a\ e$ in 2, and draw 2-2 parallel to $e\ m$, meeting $e\ f$ in 2 intersecting the arc $a\ c\ e$ in c. From the point 2 in $e\ f$ draw $2\ h$ perpendicular to $e\ f$, and make $2\ h$ equal to $2\ c$. Through the three points e, h, f, describe the arc $e\ h\ f$. Draw $f\ m$ perpendicular to $e\ f$, and prolong 2-2 to meet $f\ m$ in 2. Draw $2\ k$ perpendicular to $f\ m$; make $2\ k$ equal to $2\ c$, and through the three points f, k, m, describe the arc $f\ k\ m$.

In any convenient place parallel to $O\ E$ draw $x\ z$ (Fig. 3), and in $x\ z$ make $x\ y$ equal to half the radius of the cylinder. Draw $y\ w$ perpendicular to $A\ O$ (Fig. 1), and draw $x\ w$ perpendicular to $O\ E$. Make $y\ z$ equal to half the breadth of the beds; join $w\ z$, and $y\ w\ z$ is the angle of the twist.

ANOTHER METHOD OF FORMING THE ARCH-STONES.

The developement fig. 1, *and the templets No.* 1, *No.* 2, *and No.* 3, *fig.* 2, *being constructed as explained in the last problem.*

Let *p q r s* be the developement of the soffit of one of the stones Find the centre *o* by drawing the two diagonals *q s*, *r p*. Through *o* draw *t u* parallel to *A G*, meeting *p q* in *t*, and *s r* in *u*, and *t u* is a line parallel to the axis of the cylinder. Let *P Q R S* (Fig. 3) represent that face of the stone which is to be the soffit. With a straight edge No. 4, cut a chisel draught *T U* through the centre of this face making the angle *P T U* equal or nearly equal to the angle *p t u*, which the bed lines make with the axis of the cylinder, so that the bottom of the chisel draught may be parallel to the face of the stone, and of sufficient depth to allow for forming the concave surface of the cylinder to the full extent. Make two templets *a b c*, *a b c*, No. 5, No. 6, equal portions of the arc *A B C*, and let the points *b*, *b* be the middle points of the circular edges. Perpendicular to *T U* draw *T V*, *U W*. Cut two chisel draughts under *T V*, *U W*, in such a manner that when the circular edges of the two templets No. 5 and No. 6, are applied upon each of these draughts. and the middle points *b*, *b* upon *T* and *U*, and if the straight edges *a c*, *a c* are out of winding, and parallel to the surface of the stone, and if the bottoms of the draughts are sunk to the depth of the straight draught, and if the circular edges of the templets coincide with the bottoms of the two draughts, the bottoms of these two draughts shall be portions of the cylindric surface or of the surface of the soffit to be formed. By means of the curved edge *e i h g f* of the templet No. 1, run two draughts along each margin of the stone adjacent to the bed lines to the same depth as the bottoms of the straight and circular draughts; and by the templet No. 2, run two draughts along the ends adjacent to the joint lines to the same depth as the other two. The superfluous stone between the draughts being cut away will form the cylindric surface of the soffit.

Having made a limber mould* to the developement *p q r s*, and having drawn the line *t u* upon this mould, press the mould upon the concave surface thus formed for the soffit, so that all the points may be in contact with the cylindric surface, and that the line *t u* upon the mould may fall upon *T U* on the cylindric surface of the stone. In this state draw four lines round the four edges of this mould upon the concave surface of the stone. The two longest of these lines will be the bed lines, and the two shortest the joint lines. The beds of the stone are formed by means of the arch square No. 3, by applying the circular edge to the soffit. The other arch square for forming the joints is not exhibited.

The reason of the construction and application of these moulds explained as above, as also in Problem V., page 8, will be made more evident by examining the explanation to Problem VII., page 12.

* Zinc is well adapted to this purpose.

A second example with a different method of finding the developement of the curve which is the intersection of the intrados and the plane of the face of the arch, as also the method of finding the position of the bed lines, so that the arch stones may be equally thick.

Let $A\ H\ Q\ G$ be the plan of the arch, $A\ H$, $G\ Q$ being the representation of the faces, and let the arc $A\ B\ C$ be the right section, $A\ C$ the chord being perpendicular to $A\ G$. Divide the arc $A\ B\ C$ into two equal parts at B, and draw $B\ J$ perpendicular to $A\ C$, meeting $A\ H$ in J. Through J draw $d\ K$ parallel to $C\ A$ meeting $H\ Q$ in K. Prolong $G\ A$ to meet $d\ K$ in L, and make $L\ d$ equal to the length of the arc $A\ B\ C$. Through d draw $E\ F$ parallel to $A\ G$. Make $d\ E$ equal to $K\ H$, and $E\ F$ equal to $A\ G$. Join $A\ E$, and draw $G\ F$ parallel to $A\ E$. Divide the arc $B\ C$ into any number of equal parts as here into four, and through the points 1, 2, 3 &c., draw lines parallel to $B\ J$, meeting $J\ H$ at a', b', c', &c., and intersecting $J\ K$ at a, b, c, &c. Bisect $L\ d$ in o. Divide $o\ L$, $o\ d$ each into the same number of equal parts into which the arc $B\ C$ is divided, viz., each into four at 1, 2, 3, &c. Draw $1\ \alpha$, $2\ \beta$, $3\ \gamma$, &c., on each side of the middle point o, and make $1\ \alpha$, $2\ \beta$, $3\ \gamma$, &c., respectively equal to $a\ a'$, $b\ b'$, $c\ c'$, &c. From the point o and through the points α, β, γ, &c. on each side of it draw the curves $o\ \alpha\ \beta\ \gamma \ldots A$, $o\ \alpha\ \beta\ \gamma \ldots E$, and the whole curve $A\ E$ is the developement of the cylindric surface and the plane of the face of the arch. Draw the curve line $G\ F$ so as to be identical to the curve line $A\ E$. Divide the straight line $E\ A$ into as many equal parts as there are ring stones in the face of the arch as here into nine, and let s, u be respectively the fourth and fifth points of division from E. Draw $F\ v$ perpendicular to $A\ E$ meeting it in v, and as the point v falls between the fourth and fifth points, but nearer to the fifth u than to the fourth s, therefore join $F\ u$. Divide each springing line $E\ F$, $A\ G$, therefore, into five equal parts, viz., $E\ F$ at the points $N, P, R\ T$, and $A\ G$ at O, Q, S, U, and join $N\ O$, $P\ Q$, $R\ S$, $T\ U$. Through the points N, P, R, T, draw lines parallel to $F\ u$ to meet the curve line $E\ A$; through the points O, Q, S, U, draw lines also parallel to $F\ u$ to meet the curve line $G\ F$, and through the intermediate points of division in $u\ A$ still parallel to $F\ u$, draw lines to meet the curves $A\ E$, $G\ F$; then $F\ u$ and all the lines parallel to $F\ u$ are the developements of the bed lines, and the alternate portions of the lines $N\ O$, $P\ Q$, &c., are the developements of the joint lines.

The templets No. 1, No. 2, No. 3, might have been drawn as before explained, or having drawn $a\ e$, $e\ f$, $f\ m$ and the arc $a\ c\ e$ as in Prob. V.; bisect the straight lines $a\ e$, $e\ f$, $f\ m$; through the point of bisection in $a\ e$ draw an ordinate, and from the points of bisection in the straight lines $e\ f$, $f\ m$, draw perpendiculars to the lines; make the height of each of these perpendiculars equal to the ordinate upon $2\ a$ (then by Prob. VII., page x, Introduction) through the three points e, h, f, describe the arc $e\ h\ f$, and through the three points f, k, m, describe the arc $f\ k\ m$. These arcs do not practically differ from the curves found in Prob. V., which curves are portions of ellipses.

PROBLEM VI.

To form the springers of an oblique arch.

Let *Q A B* (Fig. 1) be half the right section of the arch as shown in plate 24, *A Q* being a section of the face of the abutment, *A B* the section of half the intrados, *c d A e p j* a section of one of the springers, the springers being set upon a level bed, generally six inches below the springing line of the soffit or intrados. The bed line upon which they are placed is represented in section by *c d*. Figure 2 represents the inside face of the abutment as supporting the springers, *f g h i k* exhibits the concave face of one of the springers corresponding to *F G H I K* (fig. 3), *f g i k* being the part which is in the same plane with the face of the abutment, and the triangular surface *g h i* part of the soffit or intrados, each being one-fourth part of the developement of one of the stones exhibited in plate 24, where the springers are also exhibited upon the springing lines *A G*, *E F*.

The concave surface *g h i* will be ascertained by the crooked templet *c d A e* of which *c d* is a section of the bed, *d A* a section of the plane surface of the face of the plane part of the springer, and *A e* a portion of the right section of the intrados. This templet is used by applying the straight edge *c d* upon the bed marked *f k* (Fg. 2) which, from the nature of orthographical projection cannot be seen as a surface, but as a line only; this will easily be understood by the workman. While applying the straight edge *c d* of the templet upon the bed, the straight part *d A* must be applied upon the narrow face *f g i k*, and the convex edge *A e* upon the concave surface *g h i* which being formed, the bed line *h i*, and the joint line *h g* must be drawn by bending a limber mould made to the triangle *G H I* (fig. 3) which is half the developement of half of one of the stones exhibited in plate 24.

The triangular soffit of the springers being thus formed; we now proceed with the upper bed as represented by the quadrilateral figure *h m n i* (fig. 2), and the end or abutting joint by the figure *g h m l*.

We may now suppose that the templets have been constructed as in plate 24, and that the arch square No. 3 is that for working the beds when the soffit is given or finished or for working the soffits when the beds are given or finished. Then if the bed *h m n i* be so formed that while applying the arch square No. 3, so that the point *r* between the curved and straight edges may be upon the line *h i*, the curved edge *v r* upon the soffit, and the straight edge *r s* upon the upper bed, and the plane containing the two branches perpendicular to the bed line *h i*, and if the straight edge coincide with the surface the upper bed will be formed as required.

PROBLEM VII.

Having the projection or plan of a stone to explain the nature of the cylindric surface of the intrados, the properties of the spiral surfaces of the beds and joints.

Let $O\ P\ Q\ R$ (Fig. 1) be the developement of the intrados or soffit of a stone, $o\ p\ q\ r\ s\ t\ u\ v$ the entire projection of the stone, $o\ p\ q\ r$ representing the intrados, $u\ v\ s\ t$ the extrados, $q\ r\ s\ t$ the upper bed, $p\ o\ v\ u$ the lower bed, $p\ q\ t\ u$ the end of the stone which forms one side of the joint, the other side being formed by the end of the adjacent stone which must be supposed to be previously set, and $r\ o\ v\ s$ the side of the joint against which the end of the next stone is to come. Also let No. 1, No. 2, No. 3, No. 4 (Fig. 2) be the templets for working the stone, found as explained in (Pob. V page 7), and the angle of the twist $Y\ W\ Z$, found as explained (at the top of page 8); observing that if there is a want of room, the operation may be condensed so as to occupy less space, by using equal portions of the radius, and the breadth of the beds, as here by making $x\ y$ half the radius, and $y\ z$ half the breadth of the bed; then the angle $y\ w\ z$ shall be equal to the angle $Y\ W\ Z$. The templet No. 5 is a straight edge of equal breadth, and No. 6 a straight edge with tapering edges inclined to each other making an angle equal to the angle of the twist.

With regard to the curvature of the different faces of the stone all the sections of the intrados represented by $o\ p\ q\ r$ made by planes through the axis of the cylinder are straight lines, and all the sections made by planes perpendicular to the said axis, are equal arcs having the same radius as that of the cylinder or that of the right section. In every other direction the section made by a plane passing through a straight line perpendicular to the axis is a concave curve or a portion of an ellipse at the extremity of the semi axis minor, which is equal to the radius of the cylinder. The templet, or convex rule No. 1, shall coincide with the convex surface $o\ p\ q\ r$ along the marginal or bed lines $o\ p$, $r\ q$, and in every direction between and parallel to these lines, and the templet or convex rule No. 2 shall coincide with the marginal lines $p\ q$, $o\ r$, and with all lines parallel to $p\ q$, $o\ r$.

In the side $q\ r\ s\ t$ which is the representation of the upper bed of the stone, there can be no sections of the surface made by a plane that are absolutely straight lines, excepting those in which the cutting planes are perpendicular to the axis, as the lines $q\ t$, $r\ s$, the bed lines $r\ q$, $s\ t$ are very nearly straight being curves of contrary flexure, the radius of curvature in the middle being infinitely great; therefore the tangent will very nearly coincide with the curve (see the sections Fig. 1 plate 9, and explanation page xxii Introduction). It is upon this principle, however, that the stone cutter is enabled to work the bed independent of the soffit: as the bed of the stone is not only straight in lines perpendicular to the axis, but also nearly straight in lines made by the sections of planes which are parallel to the axis, so that practically speaking, two straight lines may be drawn through any given point in the spiral bed perpendicular to each other, the one being perpendicular to the axis.

Because the arch under consideration is right handed, the two lines *r q*, *r s*, being considered as in the same plane, it will require an angle to be made with a straight line *s t* at the point *s* equal to the angle *Y W Z*, or *y w z*, in order to raise the line *s t* to the plane *s r q*, or the point *t* as much as the broader end of the tapering straight edge No. 6 exceeds the narrower end; therefore if the spiral bed be cut by a plane parallel to a plane between the lines *r s*, *r q*, the section of the surface shall be convex, and if cut by a plane parallel to a plane between the lines *q r*, *q t*, the section shall be concave; hence a section of the bed made by a plane passing through any point in the bed line *r q* parallel to the sectional line *E C* of the face of the arch shall be concave. It is on this account that the joints between the ring stones of the face of an oblique arch are concave to the axis minor of the ellipse.

The arch square *v q r s* No. 3 is applied to every point of the arris *r q* as shown to work the soffit of the stone when the bed is previously wrought, or the bed of the stone when the soffit is wrought, and the arch square *t n o p* No. 4 is applied to work the ends of the stone as shown, the soffit and the beds being wrought, the two limbs of each arch square being in a plane perpendicular to the arris.

PROBLEM III.

Supposing the soffit of the arch stones to be cut into two rings of equal thickness, and the extrados required, to find the development of the extrados of the upper ring, so that the development of the extrados may have the bed and end joints at right angles to each other, and to find the templets for describing the form of the stones.

Let the right section of the intrados be the arc $A\,B\,C$, and let the arc $a\,b\,c$ be the section which divides the arch stones into two equal thicknesses. Produce $C\,A$ to D, and make $A\,D$ equal to the length of the arc $A\,B\,C$. Draw $A\,G$ perpendicular to $A\,D$, and make $A\,G$ equal to the length of the abutment. Through D draw $E\,F$ parallel to $A\,G$. Make $D\,E$ equal to $D\,E$, and $E\,F$ equal to $A\,G$. Join $A\,E$ and $G\,F$, and draw the curve lines $A\,E$ and $G\,F$ as in the former examples. Bisect $A\,D$ in h, and draw $h\,i$ perpendicular to $A\,D$, meeting the straight line $A\,E$ in i. Produce $h\,D$ to d, and make $h\,d$ equal to half the length of the arc $a\,b\,c$. Through d draw $e\,f$ parallel to $E\,F$, and $i\,k$ perpendicular to $A\,E$ meeting $e\,f$ in k. Draw $k\,j$ perpendicular to $E\,F$ meeting $E\,F$ in j. Produce $i\,k$ to j; make $k\,j$ equal to $i\,k$, and join $i\,j$. Produce $A\,E$ to e, and $F\,E$ to l. Draw $e\,l$ perpendicular to $F\,E$, and join $e\,l$. Divide the straight line $A\,E$ into as many equal parts as there are ring stones, suppose nine; therefore divide $A\,E$ into nine equal parts. Draw $F\,K$ parallel to $e\,l$, and let the point K fall upon one of the points of division, supposing that $K\,E$ contains five of the nine equal parts; therefore each abutment $A\,G$, $E\,F$ will have five springers. Divide $A\,G$, $E\,F$ each into five equal parts, viz. $A\,G$ at the points O, Q, S, U, and $E\,F$ at the points N, P, R, T. Through the points A, O, Q, S, U, draw lines parallel to $K\,F$ to meet the curve line $G\,F$; through the points N, P, R, T, draw lines also parallel to $K\,F$ to meet the curve line $A\,E$, and through the points of division in $A\,K$ draw lines again parallel to $K\,F$ to meet the curve line $G\,F$, and $K\,F$ is also; all the lines which are parallel to $K\,F$ are the developements of the bed lines upon the intrados or soffit of the arch. If the lines $N\,O$, $P\,Q$, $R\,S$, be drawn they will be parallel to $E\,A$; we will suppose them to be drawn. Bisect the parts of these lines between every other two; and through each point of bisection draw a line parallel to $e\,l$ between the two adjacent bed lines, and the figure $A\,E\,F\,G$ as shewn will be the developement of the intrados answering to the middle developement in which that of the stones are rectangles.

Draw $a\,e$ (Fig. 2) parallel to $A\,C$; with $a\,e$ as a chord and with $I\,A$, the radius of the arc $A\,B\,C$, describe the arc $a\,c\,e$. Draw $a\,f$ parallel to $A\,G$, and $e\,f$ parallel to $K\,F$. Draw $f\,m$ parallel to $l\,g$, and draw $e\,m$ parallel to $A\,G$. Bisect $a\,e$, $e\,f$, $f\,m$ each at the point 2. Draw 2 c perpendicular to $a\,e$, 2 h perpendicular to $e\,f$, and 2 k perpendicular to $f\,m$. Make 2 h, 2 k each equal to 2 c. Describe the arcs $e\,h\,f$, $f\,k\,m$; then No 1, No. 2, No. 3, are the templets for working the stones as before shown.

PROBLEM IX.

To find the curved bevels for cutting the quoin heads of an oblique arch.

Let ABC (Fig. 1) be the right section. Draw CE perpendicular to AC, and draw the straight line AE making the angle CAE equal to the complement of the angle of obliquity. Find the oblique section ADE of the cylinder (as in Prob. XIV., page xvii Introduction), and the curve ADE is the common section of the concave cylindric surface and the plane of the face of the arch. Divide the arc ABC into as many equal parts as the ring stones are in number, and through the points of division draw $b\,h$, $c\,i$, $d\,j$, &c., perpendicular to the curve line ADE.

Parallel to the chord line AE draw the straight line GL (Fig. 2) the points G, L being taken at any convenient distance from each other. Make the angle GLM equal to the acute angle CEA which the axis of the cylinder make with the sectional line, or chord. Draw GH, GI, GJ, &c., respectively parallel to $b\,h$, $c\,i$, $d\,j$, &c. (Fig. 1); draw LH, LI, LJ, &c., respectively perpendicular to GH, GI, GJ, &c., and the lines LH, LI, LJ, &c., will be respectively parallel to tangents to the curve ADE at the points b, c, d, &. (Fig. 1.)

In GL make $G\,a$, $G\,b$, $G\,c$, &c., respectively equal to GH, GI, GJ, &c., and join $a\,M$, $b\,M$, $c\,M$, &c. Then the angles $G\,a\,M$, $G\,b\,M$, $G\,c\,M$, &c., shall be the acute angles which tangent planes to the curved surface of the cylinder make with the plane of the oblique face ADE at the points b, c, d, &c. (see Prob. XXXVII., page lii Introduction).

Let Figure 3 be the curve of the bed lines (found as in No. 1 Prob. V., page 7). Then in each of the bevels No. 1, No. 2, No. 3, &c., the part $F\,h\,i$ is identical to the half $f\,h\,i$ figure 3, the straight side RF of each of the bevels being parallel to GL (Fig. 2), and the inner edges $h\,q$, $h\,s$, $h\,u$, &c., respectively parallel to $a\,M$, $b\,M$, $c\,M$, &c. (Fig. 2). The curved edge $h\,F$ applies upon the bed lines which are concave, and either of the straight edges of the parallel side serves for drawing a line across the bed, which, when the stone is set, shall be in the plane of the face of the arch.

The bevels No. 1, No. 2, No. 3, &c., are those which apply upon the bed lines where the soffit or intrados forms obtuse angles with the face of the arch, and the supplements of these bevels or angles apply upon the bed lines where the intrados forms acute angles with the said face.

Let us now suppose that the arch stones are all set except those which are required to form the quoin heads, and that the stones which are to form the quoin heads have been wrought except the heads or ends which are to form the face of the arch. Then, in order to find how much is to be cut off each stone in every course in order to complete that course; measure the distance of the remaining part of each bed line upon the developement from the end of the last stone in that

[illegible] and apply the [illegible] from the end [illegible] upon each bed [illegible] face of the arch. [illegible] that course, [illegible] line with the [illegible] other side being [illegible] The super-fluous [illegible] way between the two lines [illegible] plane of the face of the arch.

It must here be observed that by the principle of the trihedral, the [illegible] perpendicular to the curve which [illegible] the plane of the face; [illegible] surfaces, the joints on the [illegible] lines, and which do [illegible] equal parts; however [illegible] execution, very little correction will be [illegible] necessary, the method being a near approxi-mation. Its simplicity is some compensation for its introduction.

The [illegible] oblique arch at Gateshead, were [illegible] obliquity being 76° 12', the work was conducted by Mr. John [illegible]. The bevels of the stones after the first cutting wanted no correction.

Another method of forming the quoin heads derived from the principle of the spiral construction.

Figure 1 exhibits the right section, as also the oblique section or elevation, together with the developement of the intrados; figure 2 the developement of the extrados, and figure 3 the templet of the curvature of the bed lines. The oblique section is found as in Prob. XIV, page xvii, Introduction, and the two developements as in Prob. IV, page 6.

The lines in the oblique section which represent the joints in the beds, are found in the following manner. Let h s v r...I (Fig. 1) be the curve line which terminates one end of the developement of the intrados corresponding to the sectional line *H I* or *J a'* on the plan. Prolong *C A* to meet h *F* in *D*, and through the points s, v, r, &c., which are the ends of the developements of the bed lines draw s d, v d, r d, &c , parallel to h *F* intersecting the bed lines respectively in u, q, l, &c. Let the developement of the first or shortest bed line s k meet h *F* in *K*. Then because s k, v u, r q, &c., are parallel and equi-distant, the parallel lines h k, s u, v q, &c., are equal to one another. From *C* (Fig. 1) upon *C H* set off distances respectively equal to d s, d v, d r, &c. ; from the points of section in *C H* draw lines parallel to *C A* to meet the sectional line *H I* of the oblique face, and from the points of meeting draw lines perpendicular to *H I* meeting the lower curve *I M H* in the points s, v, r, &c. In the developement of the extrados (Fig. 2), the points *b*, *c*, *e*, &c., are the ends of the bed lines which terminate in the curve. Perpendicular to *a d* draw *b i*, *c i*, *e i*, &c., meeting *a d* in *i*, *i*, *i*, &c. Upon *c' J* (Fig. 1) from *c'* set off distances respectively equal to *b i*, *c i*, &c. (Fig. 2). From the points thus marked off, draw lines parallel to *c' a* or *C A* to meet the sectional line *J a'*, and from the points of meeting draw lines perpendicular to *J a'* meeting the curve of the extrados in the points *S'*, *V'*, *R'*, &c. Join *S'* s, *V'* v, *R'* r, &c., which are the representation of the intersections of the beds in the oblique section of the arch.*

Let Figure 4 be the representation of a portion of the end of the cylinder adjacent to one of the oblique faces which is represented by the curve *H S V R...I*, and which by hypothesis is identical to the semi-ellipse *H* s v r...*I* (Fig. 1). Let *S K*, *V U*, *R Q*, &c. (Fig. 4), represent portions of the first, second, third, &c , of the spiral bed lines; and let *H K*, *S U*, *V Q*, &c., be supposed to be drawn on the surface of the cylinder parallel to the axis, and let *H Y*, *S T*, *V P*, &c., be parallel to *H I*. Then the straight lines *H K*, *S U*, *V Q*, &c., are by hypothesis equal to h k, s u, v q, &c. (Fig 1), and are consequently equal to one another; moreover the angles *K H Y*, *U S T*, *Q V P*, &c., are each equal to the angle of obliquity *C H I* (Fig. 1).

* The reader must observe that the lines *S* s, *V* v, *R* r, &c., which represent the joint lines between the ring stones are not absolutely straight lines, but curves.

Prolong S s (Fig. 1) to y, T t to u, R r to p, &c., and draw s t, t u, r p, &c. parallel to H I. Let K s meet H I in y; and let H Y, U T, Q P, &c. (Fig. 4) be supposed to be made respectively equal to H y, s t, v p, &c. (Fig. 1), and let S Y, U T, R P, &c. (Fig. 4) be joined; then by hypothesis S Y, U T, R P, &c., are respectively equal to s y, t v, p r, &c. (Fig. 1). Without repetition it may be recollected that in each of the triangles represented by *K H Y*, *U S T*, *Q V P*, &c. (Fig. 4) two sides and the contained angle are given; therefore in each respective triangle the third side may be found, and in each of the triangles represented by *K Y S*, *U T V*, *Q P R*, &c., the three sides are given; hence the angles represented by *K S Y*, *V U T*, *Q R P*,* &c., may be found.

It may be readily admitted that a short length of the developement of a spiral line will not differ sensibly from the length of the line itself; therefore the lines s k, v u, r q, &c. (fig. 1) which are portions of the developements of the bed lines will in practice be equal to equal lengths of the spirals themselves.

Draw *h k* (No. 1) in any convenient situation: make the angle *k h y* equal to the angle C H I (Fig. 1), and make *h k* (No. 1) equal to h k (Fig. 1). Make *h y* (No. 1) equal to H y (Fig. 1), and join *k y* (No. 1). With the distance y s (Fig. 1) from *y* (No. 1) describe an arc at s, and with the distance k s (Fig. 1) from *k* (No. 1) describe another arc intersecting the former in *s*; join *y s* and prolong *ys* to *a*.

Draw *s u* (No. 2) in any convenient situation: make the angle *u s t* equal to the angle C H I (Fig. 1), and make *s u* (No. 2) equal to s u (Fig. 1). Make *s t* (No. 2) equal to s t (Fig. 1), and join *u t* (No. 2). With the distance t v (Fig. 1) from *t* (No. 2) describe an arc at v, and with the distance u v (Fig. 1) from *u* (No. 2) describe another arc intersecting the former in *v*; join *t v*, and prolong *t v* to *s*.

Draw *v q* (No. 3) in any convenient situation: make the angle *q v p* equal to the angle C *H I* (Fig. 1), and make *v q* (No. 3) equal to v q (Fig. 1). Make *v p* (No. 3) equal to v p (Fig. 1), and join *q p* (No. 3). With the distance p r (Fig. 1) from *p* (No. 3) describe an arc at r, and with the distance q r (Fig. 1) from *q* (No. 3) describe another arc intersecting the former in *r*. Join *p r*, and prolong *p r* to *y*.†

With the mould of curvature of the bed lines draw the curve line *k s* (No. 1), the curve line *u v* (No. 2), the curve line *q r* (No. 3), &c.,

* The angles represented by *K S Y*, *V U T*, *Q R P*, &c., are respectively contained by the curve lines *K S*, *U V*, *Q R*, &c., and the straight lines *S Y*, *V T*, *R P*, &c., and therefore represent the complements of the angles made by the bed lines on the soffit and on the face or elevation of the arch.

† The reader will no doubt have observed that the lines *h k* (No. 1), *s u* (No. 2), *v q* (No. 3), must be equal to each other, because the lines h k, s u, v q, &c. &c., (developement of the intrados) are equal to each other, and that the lines *h y* (No. 1), *s t* (No. 2), *v p* (No. 3), &c., are respectively equal to h y, s t, v p, &c. (oblique section); moreover, that the *y s* (No. 1), *t v* (No. 2), *p r* (No. 3), &c., are respectively equal to y s, t v, p r, &c. (oblique section), and that the lines *k s* (No. 1), *u v* (No. 2), *q r* (No. 3), &c., are respectively equal to k s, u v, q r, &c., (devolopement of the intrados).

then the angle $k\,s\,\alpha$ (No. 1) contained by the curve line $k\,s$ and the straight line $s\,\alpha$ is the bevel adapted to the first joint; the angle $u\,v\,\beta$ (No. 2) contained by the curve line $u\,v$ and the straight line $v\,\beta$ is the bevel adapted to the second joint; the angle $q\,v\,\gamma$ (No. 3) contained by the curve line $q\,r$ and the straight line $r\,\gamma$ is the bevel adapted to the third joint, and so on.

In each case the curved edge must be applied along the arris of the stone between the soffit and bed, and the straight edge across the bed, in order to draw the line which, when the stone is cut, the surface shall be in the plane of the face.

The above [illegible]
heads is by [illegible]
approximate, [illegible]
and [illegible]
demonstra[illegible]
in this [illegible]
more judg[illegible]
in the [illegible] this [illegible]
chaos of curves & [illegible]
confusion [illegible]
safe [illegible]
error. [illegible]

[illegible]

[illegible] straight edges [illegible] elevation of [illegible] Prepare two [illegible] No. 1, represent- [illegible] Make [illegible] and these [illegible] upon the soffit. [illegible] so as to [illegible] the section No. [illegible] exactly to [illegible] mould No. 4 so [illegible] and the side [illegible] perpendicular to [illegible] No. 4, so that the [illegible] the plane of [illegible] with the straight edge [illegible] along the straight [illegible] drawn will make [illegible] equal to the angle [illegible] with the bed [illegible]

[illegible] range in the plan of [illegible]

[illegible] stones, as made [illegible]

[illegible] is a spiral surface and [illegible] therefore a near approxi- [illegible]

ON THE THEORY OF THE OBLIQUE ARCH;

FROM WHICH, BY HAVING THE DIMENSIONS GIVEN, MAY BE FOUND THE RADII OF CURVATURE OF THE JOINT AND BED LINES ON THE INTRADOS, AND ALL THE NECESSARY LENGTHS OR DISTANCES OR ANGLES REQUIRED IN THE CONSTRUCTIONS INDEPENDENT OF THE DEVELOPEMENT.

Proposition 1.

To find the developement of the curve which is the section of the segment of a cylinder made by a plane, cutting the cylinder obliquely to the axes; given the radius of the cylinder, the chord and height of the arc of the right section, and the distance of obliquity.

Let B (Fig. 1) be the middle of the arc $A\ B\ C$, which is the right section of the cylindric surface, $A\ C$ being the chord. Prolong $C\ A$ to D, and make $A\ D$ equal to the length of the arc $A\ B\ C$. Draw $C\ H$, $D\ E$, each perpendicular to $C\ D$, making each equal to the distance of obliquity. Join $A\ H$, and draw $B\ O$ perpendicular to $A\ C$, meeting $A\ H$ in O. Through O draw $K\ d$ parallel to $C\ D$, meeting $C\ H$ in K and $D\ E$ in d, and draw $A\ L$ parallel to $C\ H$ or $D\ E$, meeting $K\ d$ in L. Bisect $L\ d$ in o, and $o\ L$, $o\ d$ are each equal to the length of the arc $B\ A$ or $B\ C$.

Let $B\ u$ be any indefinite part of the arc $B\ C$. Draw $u\ w$ parallel to $B\ O$, meeting $O\ H$ in w and intersecting $O\ K$ in v. In $o\ d$ or $o\ L$ make $o\ x$ equal to the length of the arc $B\ u$, and draw $x\ y$ perpendicular to $o\ d$. Make $x\ y$ equal to $v\ w$, and y is a point in the developpment of the curve; or by calculation thus:—

Let V be the centre of the arc $A\ B\ C$. Draw $u\ \alpha$ parallel to $C\ A$, meeting $B\ O$ in α.

Let $b = o\ L$ or $o\ d$, which is the length of the arc $B\ A$ or $B\ C$,
$x = o\ x$ the length of the indefinite arc $B\ u$,
$y = v\ w = xy$,
$e =$ the measure of the arc $B\ C$ or $B\ A$ in degrees, &c,
$v =$ the measure of the arc $B\ u$ in the same parts,
$m =$ the chord $A\ C$,
$n = C\ H$ the distance of obliquity,
and $R =$ the radius $V\ u$.

$b : x :: e : v = \frac{e\,x}{b}$ which is the measure of the angle $u\ V\ B$. But in the triangle $u\ V\ \alpha$ right angled at α, we have the angle $u\ V\ \alpha = \frac{e\,x}{b}$, and the side $V\ u = R$, to find the side $u\ \alpha$.

By Trigonometry,— $1 : \sin.\frac{e\,x}{b} :: R : s\,w = O\,v = \left(\sin.\ \frac{e\,x}{b}\right)R.$

By similar triangles $A\,C\,H$, $O\,v\,w$:

$A\,C : C\,H :: O\,v : v\,w$;

or $m :: n :: \left(\sin.\ \frac{e\,x}{b}\right)R : v\,w = \frac{n}{m}\left(\sin.\ \frac{e\,x}{b}\right)R$;

but $v\,w = x\,y = y$: therefore $y = \frac{n}{m}\left(\sin.\ \frac{e\,x}{b}\right)R.$

To draw a tangent to the curve at the point of retrogression.

Join $C\,O$; through o draw $F\,G$ parallel to $C\,O$, and $F\,G$ shall be a tangent to the curve at o.

If the values of x increase by unity, the values of y are a series of sines continually decreasing, showing that the curve is concave towards the axis; hence the point o is the point of retrogression. But besides this method which is easily understood; a more general one is to differentiate the ordinate a second time, and if its value be negative, the curve is concave towards the axis: but if positive, convex.

The geometrical construction of the problem has been shown, Introduction (Prob. xv., page xviii.)

PROPOSITION 2.

Given the radius of the cylinder, the length and breadth of a spiral; to find its projection spiral.

Let B (Fig. 2) be the middle point of the arc $A\,B\,C$, and let the breadth of the spiral be equal to the length of the arc $A\,B\,C$.

Draw $B\,D$ perpendicular to $A\,C$, and make $B\,D$ equal to the height of the spiral. Bisect $B\,D$ in O, and in $O\,B$ or $O\,D$ from O take $O\,x$ any indefinite part of $O\,B$. Draw $x\,y$ perpendicular to $O\,B$, and take the arc $B\,u$ the same part of the arc $B\,C$ which the straight line $O\,x$ is of the straight line $O\,B$ Draw $u\,z$ parallel to $C\,A$ meeting $B\,V$ in z. Make $x\,y$ equal to $z\,u$ and y is a point in the curve.

Or, by calculation, thus:—

Let $a =$ the straight line $O\,B$ or $O\,D$,

$e =$ the measure of the arc $B\,A$ or $B\,C$ in degrees, &c.

$x = O\,x$ any indefinite part of $O\,B$,

and $y = zu = xy$,

$$a : x :: e : \angle B\,V\,u = \frac{e\,x}{a}.$$

Then in the triangle $u\,z\,V$ right angled at z are given, the side $V\,u = R$, and the $\angle\,z\,V\,u = \frac{e\,x}{a}$, to find the side $z\,u$;

By trigonometry $1 : \sin.\frac{e\,x}{a} :: R : z\,u = \left(\sin.\ \frac{e\,x}{a}\right).\ R$;

hence $y=\left(\sin. \frac{e\,x}{a}\right). R$, which is the equation of the curve.*

To draw a tangent to the curve at O.

Draw $B\,F$ perpendicular to $B\ O$, and make $B\ F$ equal to the length of the arc $B\ C$. Join $F\ O$, and prolong $F\ O$ to G. Then $F\ G$ is a tangent to the curve at O, or $F\ G$ is the developement of the spiral of which $P\ Q$ is the projection.

Proposition 3.

To find the section of a spiral surface cut by a plane parallel to the axis of the cylinder.

Let B (Fig. 3.) be the middle point of the arc $A\ B\ C$, and let the breadth of the spiral be equal to the length of the arc $A\ B\ C$.

Draw $B\ D$ perpendicular to $A\ C$, and make $B\ D$ equal to the height of the spiral. Bisect $B\ D$ in O, and in $O\ B$ from O take $O\ x$ any indefinite part of $O\ B$. Draw $x\ y$ perpendicular to $O\ B$, and take the arc $B\ \alpha$, the same part of the arc $B\ C$, which the straight line $O\ x$ is of the straight line $O\ B$. Draw $B\ P$ parallel to $A\ C$, and prolong $V\ \alpha$ to meet $B\ P$ in u. Make $x\ y$ equal to $B\ u$, and y is a point in the curve.

Let $a=$ the straight line $O\ B$ or $O\ D$,
$x=O\ x$, any indefinite part of the straight line $O\ B$,
$y=B\ u=x\ y$,
and $e=$ the measure of the arc $B\ A$ in $B\ C$ in degrees, &c.

$$a : x :: e : \angle\ B\ V\ u=\frac{e\,x}{a}$$

Then in the triangle $V\ B\ u$ right-angled at B are given—

The side $V\ B=R$, and the angle $B\ V\ u=\frac{e\,x}{a}$, to find $B\ u$;

By trigonometry $1 : \tan. \frac{e\,x}{a} :: R : B\,u=\left(\tan. \frac{e\,x}{a}\right)R$,

Hence $y=\left(\tan. \frac{e\,x}{a}\right). R$, which is the equation of the curve.

To draw a tangent to the curve at O.

Draw $B\ F$ perpendicular to $B\ O$, and make $B\ F$ equal to the length of the arc $B\ C$. Join $F\ O$ and prolong $F\ O$ to G; then $F\ G$ is a tangent to the curve at O. The curve $P\ O\ Q$ is a section of the spiral surface of the bed of a stone.

* Thus let $e=60$, and x be respectively equal to 1, 2, 3, 4; then the arcs $B\ u$, $B\ v$, $B\ w$, $B\ C$ will be respectively 15°, 30°, 45°, 60°, and the values of y will be respectively (sin. 15°)· R, (sin. 30°)· R, (sin. 45°)· R, (sin. 60°)· R.

[illegible]

[illegible]

[illegible] may be [illegible]

[illegible]

We shall now proceed [illegible] calculations which will be found necessary [illegible] oblique arch.

[illegible] the [illegible] being the length of [illegible] the angles [illegible] perpendicular [illegible] meeting [illegible] and [illegible] the distance between the [illegible] lines.

The [illegible] is the development of the courses, A [illegible] $E F$ [illegible] perpendicular [illegible] contained between [illegible] is equal to the angle [illegible] the arc [illegible] B [illegible]. The [illegible] in which the joints are arranged. Draw [illegible] perpendicular [illegible] meeting A [illegible] in [illegible] is the breadth of each of the springing courses, moreover $A F$ is the breadth of each of the subsequent courses. It is evident in the developement, that since the bed lines are all perpendicular and the joint lines parallel to A [illegible], the springers are equal right-angled triangles each similar to the triangle [illegible] the triangle E [illegible] the triangle [illegible]. The construction of the developement of the [illegible] has already been explained.

EXAMPLE 1.

Oblique arch over Willington Waggonway, upon the Newcastle and North Shields Railway.

DESIGNED BY ROBERT NICHOLSON, C. E.

In this example are given the distance of obliquity equal to 13·63 feet, the width of the right section of the archway equal to 17·374 feet, the height of the intrados equal to 6 feet, the length of each springing line equal to $32\frac{1}{2}$ feet, and the number of courses equal to 23; to find the requisites for executing the work.

From the chord 17·374 feet, and the height 6 feet of the arc, will be found the radius 9·29 feet (Introduction, Page xxxiii, Prob. XXV.), and the length of the arc 22·46 feet (Introduction, Page xxxi., Prob. XXIV.).

In the triangle $A\ D\ E$ right angled at D are given $A\ D=22{\cdot}46$ feet, and $D\ E=13{\cdot}63$ feet, to find $A\ E$, and thence the radii of curvature.

$$\begin{aligned} &\text{Now } A\ D^2=22{\cdot}46^2=504{\cdot}4516=504{\cdot}45 \text{ feet nearly,}\\ &\text{and } D\ E^2=13{\cdot}63^2=185{\cdot}7769=185{\cdot}78 \text{ feet nearly.}\\ \therefore\ &A\ E^2=A\ D^2+D\ E^2=690{\cdot}2285=690{\cdot}23 \text{ feet nearly.}\\ &A\ E=\sqrt{(690{\cdot}2285)}=26{\cdot}27 \text{ feet nearly.}\end{aligned}$$

Let $r=9{\cdot}29$ feet, the radius of the cylinder, R equal the radius of curvature of the joint line spirals, R' equal to the radius of curvature of the bed line spirals.

$$\left.\begin{aligned}\text{Then } R&=\frac{AE^2\times r}{AD^2}\\ \text{and } R'&=\frac{AE^2\times r}{DE^2}\end{aligned}\right\} \text{See Introduction, page xlvi., Prob. XXIV.}$$

Now $A\ E^2\times r=690{\cdot}23\times 9{\cdot}29=6412{\cdot}24$ nearly;
therefore $R=6412{\cdot}24\div 504{\cdot}45=12{\cdot}6$ feet nearly,
and $R'=6412{\cdot}24\div 185{\cdot}78=34{\cdot}5$ feet nearly.

By similar triangles $A\ E\ D$, $F\ E\ V$.

$$A\ E : E\ D :: F\ E : E\ V=\frac{E\ D\times F\ E}{A\ E}=\frac{13{\cdot}63\times 32{\cdot}5}{26{\cdot}27}=16{\cdot}86 \text{ feet.}$$

$A\ V=A\ E-V\ E=26{\cdot}27-16{\cdot}86=9{\cdot}41$ feet.

Then as $A\ E$ is to $V\ E$ so is the entire number of courses to the number of springers, or springing courses.

Let $a=23$ be the entire number of courses, and x the number of springers required, $A\ E : E\ V :: a : x=\frac{E\ V\times a}{A\ E}=\frac{16{\cdot}86\times 23}{26{\cdot}27}=14{\cdot}76$;

but, as 14·76 is nearer to 15 than to 14, let the number of springers be 15, and the 15 springers will have 15 arch stones for the heads of the courses which they support.

Now $23-15=8$, the number of courses, independent of the springing courses.
or, $15+8=23$, the entire number of courses,
$16{\cdot}86\div 15=1{\cdot}124$ feet, for the thickness of the springing courses,
and $9{\cdot}41\div 8=1.176$ feet, the thickness of the independent courses

Example 2.

Oblique arch over High Street, Gateshead, upon the Brandling Junction Railway.

DESIGNED BY JOHN & BENJAMIN GREEN, ARCHITECTS, &c.

In this example are given the distance of obliquity equal to 9·166 feet, the width of the right section equal to 38·75 feet, the height of the intrados equal to 6 feet, the width above the arch between the outsides of the parapets equal to 26 feet, and the number of courses equal to 29.

By similar triangles $A\,C\,H$, $Q\,P\,H$.

$A\,C : C\,H :: Q\,P : P\,H.$

$$\therefore P\,H = \frac{C\,H \times Q\,P}{A\,C} = \frac{9{\cdot}166 \times 26}{38{\cdot}75} = 6{\cdot}15 \text{ feet},$$

$\therefore H\,Q = \sqrt{(P\,Q^2 + P\,H^2)} = \sqrt{(26^2 + 6{\cdot}15^2)} = 26{\cdot}71$ feet, which is the length of each springing line or abutment.

From the chord equal to 38·75 feet, and the height of the arc equal to 6 feet are found the radius equal to 34·28 feet, and the length of the arc $A\,B\,C$ equal to 41·19 feet, which is the breadth of the development of the intrados.

In the triangle $A\,D\,E$, right angled at D, are given $A\,D = 41{\cdot}19$ feet, and $D\,E = C\,H = 9{\cdot}166$ feet, to find $A\,E$.

Now $A\,D^2 = 41{\cdot}19^2 = 1696{\cdot}6161 = 1696{\cdot}62$ nearly,

and $D\,E^2 = 9{\cdot}166^2 = 84{\cdot}0155 = 84{\cdot}$ nearly.

$\therefore A\,E^2 = A\,D^2 + D\,E^2 = 1780{\cdot}6316 = 1780{\cdot}63$ nearly.

$\therefore A\,E = \sqrt{(1780{\cdot}6316)} = 42{\cdot}197 = 42{\cdot}2$ feet nearly, which is the length of the spiral joint line on the intrados.

Now $A\,E \times r = 1780{\cdot}63 \times 34{\cdot}28 = 61039{\cdot}9964 = 61040{\cdot}$ nearly.

$\therefore R = 61040 \div 1696{\cdot}62 = 35{\cdot}9 = 36$ feet, nearly,

and $R' = 61040 \div 84 = 726{\cdot}7 = 726$ feet, nearly.

By similar triangles $A\,E\,D$, $F\,E\,V$.; $A\,E : E\,D :: F\,E : E\,V$.

$$\therefore E\,V = \frac{E\,D \times F\,E}{A\,E} = \frac{E\,D \times A\,G}{A\,E} = \frac{9{\cdot}166 \times 26{\cdot}71}{42{\cdot}2} = 5{\cdot}8 \text{ feet nearly}$$

Then $A\,E : E\,V :: a : x$

$$\therefore x = \frac{E\,V \times a}{A\,E} = \frac{5{\cdot}8 \times 29}{42{\cdot}2} = 4 \text{ nearly for the number of springers.}$$

Now $29 - 4 = 25$ the number of whole courses, and $A\,V = A\,E - E\,V = 42{\cdot}2 - 5{\cdot}8 = 36{\cdot}4$ feet, which is the breadth of the developement of the intrados of the entire courses.

$\therefore 5{\cdot}8 \div 4 = 1{\cdot}45$ the thickness of the springing courses.

and $36{\cdot}4 \div 25 = 1{\cdot}456$ the thickness of the entire courses.

By similar triangles $E\,D\,A$, $E\,u\,v$.

$E\,D : D\,A :: E\,u : u\,v.$

$$\therefore u\,v = \frac{D\,A \times E\,u}{E\,D} = \frac{41{\cdot}19 \times 1.45}{9{\cdot}166} = 6{\cdot}51 \text{ feet nearly, which is the}$$

length of the inclined back of each springer, and $26{\cdot}71 \div 4 = 6{\cdot}677$ the length of the bed.

EXAMPLE 3.

Being one of the four oblique arches of the bridge over the river Tees upon the great North of England Railway, near Croft.

DESIGNED BY HENRY WELCH, C. E.

In this example are given the angle of obliquity equal to 50°, the span of the oblique face equal to 60 feet, the height of the intrados equal to 14·5 feet, the distance between the parapets 28·75 feet, and the breadth of the courses on the intrados equal to 13 inches.

In the triangle $A\ C\ H$ right angled at C are given the angle $A\ H\ C$ equal to 50°, and the hypothenuse $A\ H$ equal to 60 feet to find $A\ C$ and $H\ C$.

rad. : sin. $H°$: : $A\ H$: $A\ C$.

$\therefore A\ C = \frac{\sin. H° \times A H}{\text{rad}^*} = \sin. 50° \times 60 = \cdot 7660 \times 60 = 45\cdot 96$ feet nearly,

and rad. : cos. $H°$: : $A\ H$: $H\ C$.

$\therefore H\ C = \frac{\cos. H° \times A\ H}{\text{rad.}} = \cos.\ 50° \times 60 = \cdot 6428 \times 60 = 38\cdot 568$ feet nearly, which is the distance of obliquity.

In the right-angled triangle $H\ P\ Q$, are given in the angle $P\ H\ Q$ or $A\ H\ Q$ equal to 50°, and the opposite side $P\ Q$ equal to 28·75 feet, to find $H\ Q = A\ G = E\ F$.

sin. $H°$: rad. : : $P\ Q$: $H\ Q$.

$\therefore H\ Q = \frac{\text{rad.} \times P\ Q}{\sin.\ H°} = \frac{28\cdot 75}{\sin. 50} = \frac{28\cdot 75}{\cdot 766} = 37\cdot 53$ feet, which is the length of the springing line; hence $A\ G = EF = 37\cdot 53$ feet.

From the chord of the arc equal to 45·96 feet, and the height of the intrados equal to 14·5 feet, we shall find the radius of the circle equal to 25·46 feet nearly, and the length of the arc equal to 57·32 feet nearly.

$$\text{Now } A\ D^2 = 57\cdot 32^2 = 3285\cdot 5824 = 3285\cdot 58 \text{ nearly.}$$
$$\text{and } D\ E^2 = 38\cdot 57^2 = 1487\cdot 6449 = 1487\cdot 64 \text{ nearly.}$$
$$\therefore A\ E^2 = A\ D^2 + D\ E^2 = 4773\cdot 2273 = 4773\cdot 23 \text{ nearly.}$$
$$\therefore A\ E = \sqrt{(4773\ 2273)} = 69\cdot\ 08 \text{ nearly.}$$
$$\therefore A\ E^2 \times r = 4773\cdot 23 \times 25\cdot 46 = 121526\cdot 4358.$$
$$\therefore R = 121526\cdot 44 \div 3285\cdot 58 = 36\cdot 99 \text{ feet nearly.}$$
$$\therefore R' = 121526\cdot 44 \div 1487\cdot 64 = 81\cdot 69 \text{ feet nearly.}$$

By similar triangles $A\ E\ D$, $F\ E\ V$.

$$A E : E D :: F E : E\ V = \frac{E D \times F E}{A E} = \frac{38\cdot 568 \times 37\ 54}{69\cdot 08} = 20\cdot 95 \text{ ft.}$$

Now $69\cdot 08 \times 12 \div 13 = 63\cdot 7$ for the number of courses, say 63.

$69\cdot 08 : 20\cdot 95 :: 63 : x = \frac{20\cdot 95 \times 63}{69\cdot 08} = 19\cdot 1 = 19$ nearly, which is the number of springers in each abutment.

$\therefore 37\cdot 53 \div 19 = 1\cdot 97$ feet nearly, for the length of the bed of each springer.

$E\,u = 20\cdot 95 \times 12 \div 19 = 13\cdot 23$ inches, which is the thickness of each of the springing courses, or the breadth of each springing course on the soffit.

*Radius=1.

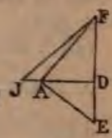
Now $63-19=44$, which is the number of entire courses, and $69\cdot08-20\cdot95=48\cdot13$ feet, is the breadth of the soffit of the entire courses.

$\therefore$ $48\cdot13\times12\div44=13\cdot12$ inches, which is the thickness of the entirre courses.

By similar triangles $E\ D\ A$, $E\ u\ v$,

$E\ D : D\ A :: E\ u \therefore u\ v$

$\therefore u\ v = \frac{D\ A\times E\ u}{E\ D}=\frac{57\cdot32\times13\cdot23}{38\cdot57}=19\cdot66$ inches, which is the length of the inclined back.



To find the angle of the twist.

In the triangle $A\ D\ E$, right-angled at D, are given $D\ E=38\cdot57$, and $A\ D=57\cdot32$; to find the angle $E\ A\ D$.

$$AD : DE :: 1 : \text{tan.}\ D\ A\ E=\frac{D\ E}{A\ D}=\frac{38\cdot57}{57\cdot32}=\cdot6728=\text{tan.}\ 33^{\circ}56'.$$

In the triangle $A\ D\ F$, right-angled at D, are given the side $A\ D=25\cdot46=$the radius of the cylinder, and the angle $A\ F\ D$ equal to the angle $D\ A\ E=36^{\circ}\ 56'$, to find $D\ F$.

$$\text{tan.}\ A\ F\ D : 1 :: A\ D : D\ F=\frac{A\ D}{\text{tan}\ \ A\ F\ D}=\frac{25\cdot46}{\text{tan.}\ 36\cdot56}=\frac{25\cdot46}{6728}=$$

$37\cdot84$ feet.

Suppose now the thickness of the arch to be 2 feet 6 inches$=2\cdot5$; then $25\cdot46+2\cdot5=27\cdot96$ the radius of the exterior cylinder.

Produce the side $D\ A$ of the triangle $A\ D\ F$ to J; make $D\ J=27\cdot96$, and join $A\ F$. Then in the triangle $J\ D\ F$, right-angled at D, are given the side $J\ D=27\cdot96$, and the side $D\ F=37\cdot84$, to find the angle $J\ F\ D$.

$$F\ D : D\ J :: 1 : \text{tan.}\ J\ F\ D=\frac{D\ J}{F\ D}=\frac{27\cdot96}{37\cdot84}=\cdot73\cdot89=\text{tan.}\ 36^{\circ}\ 37'$$

Hence $\angle J\ F\ D-\angle A\ F\ D=\angle A\ F\ J$ the angle of the twist.

or $36^{\circ}27-33^{\circ}56=2\cdot31'$ the angle of the twist.

But as measures in degrees, &c. are not adopted for the use of workmen, it will be more convenient to reduce the breadth at a certain distance from the angular point to inches.

Suppose the length of each of the winding rules to be 5 feet, and the breadth of the parallel one to be three inches; it will be

$1 : \text{tan.}\ 2^{\circ}\ 31 :: 5 :$ the additional breadth at the greater end,

or $1 : \cdot04395 :: 5 : \cdot21975$ feet$=2\cdot64$ inches nearly.

$\therefore 3+2\cdot64=5\cdot64$ inches which is the breadth of the greater end, 3 inches being that of the less end.

For the application of those straight edges, see page 8, as also pages 12 and 13, all the six templets are exhibited in plate 26, as No. 1, No. 2, No. 3, &c.

EXAMPLE FOR PRACTICE.

In this example are given the length of the oblique face, equal to 42 feet, the width of the right section of the arch-way equal to 19 feet, the height of the intrados equal to 7 feet 1 inch, and the distance between the external faces of the parapets equal to 14 feet. By similar triangles $C\,A\,H$, $P\,Q\,H$.

$$A\,C : A\,H :: Q\,P : Q\,H.$$

$\therefore Q\,H = \dfrac{A\,H \times Q\,P}{A\,C} = \dfrac{42 \times 14}{19} = 30{\cdot}9473$ feet, the length of the abutments or springing lines.

From the chord of the arc equal to 19 feet, and the height of the intrados equal to 7 feet 1 inch, will be found the radius of the circle equal to 9·912 feet, and the length of the arc equal to 25·4026 feet nearly.

Now $A\,D^2 = 25{\cdot}4026^2 = 645{\cdot}29208676 = 645{\cdot}3$ nearly.
and $D\,E^2 = 37{\cdot}4566^2 = 1402{\cdot}99688356 = 1403$ nearly.
$\therefore A\,E^2 = A\,D^2 + D\,E^2 = 2048{\cdot}28897032 = 2048{\cdot}3$ nearly.
$\therefore A\,E = \sqrt{(2048{\cdot}3)} = 45{\cdot}258$ feet nearly.
$\therefore A\,E^2 \times r = 2048{\cdot}3 \times 9{\cdot}912 = 20302{\cdot}75$ nearly.
$\therefore R = A\,E^2 \times r \div A\,D^2 = 20302{\cdot}75 \div 645{\cdot}3 = 31{\cdot}4$ feet nearly.
$\therefore R' = A\,E^2 \times r \div D\,E^2 = 20302{\cdot}75 \div 1403 = 14{\cdot}4$ feet nearly.

Then by similar triangles $A\,E\,D$, $F\,E\,V$

$$A\,E : E\,D :: F\,E : E\,V$$

$\therefore E\,V = \dfrac{E\,D \times F\,E}{A\,E} = \dfrac{37{\cdot}4566 \times 30{\cdot}9473}{45{\cdot}258} = 25{\cdot}612$ ft. the breadth of the springing courses on the head of the arch.

$\therefore$ 45—26=19, the number of ring stones, independent of the springers, and 45·258—25·612=19·645 feet, the length of the soffit of the independent springers.

$\therefore$ 30.9473÷26=1·19 feet nearly for the length of the bed of each springer.

$\therefore E\,u = 25{\cdot}612 \times 12 \div 26 = 11{\cdot}82$ inches, the breadth of the soffit of each springing course.

$\therefore$ 19·643 x 12÷19=12·406 inches, the breadths of the soffits of the independent courses.

And by similar triangles $E\,D\,A$, $E\,u\,v$.

$$E\,D : D\,A :: E\,u : u\,v$$

$\therefore u\,v = \dfrac{D\,A \times E\,u}{E\,D} = \dfrac{25{\cdot}4026 \times 11{\cdot}82}{37{\cdot}4566} = 8{\cdot}001 = 8$ inches nearly,

which is the length of the inclined back of each springer.

Observation.

To construct an oblique arch entirely of stone, is in some countries where it is difficult to procure, very expensive; however in order to build one which will be sufficiently strong at a moderate price, it is necessary that the imposts or springings should be of stone, and to have the appearance of good work the quoins which form the ring stones, and the head of the arch should also be of stone; then the intermediate parts of the courses may be of brick, allowing perhaps four courses of bricks to each stone springer, depending on thickness at the abutment. To work the springers and the quoin heads, the same templets will be required as if the arch had been constructed entirely of stone. The templets are described in pages 7, 9, 10, 11, 12, 15. Previous to setting the brick courses, the boarding or laggings should be truly adjusted and fixed, and for the regulation of the work the bed lines should be drawn thereon in their true position. In order to try the work as the bricklayer proceeds, he ought to use a kind of set square made of thin board containing an angle exactly the reverse of the templet No. 3 plate 22, and consequently the curved edge will be concave instead of being convex, as in the arch square. In trying any course the set square must have the point between the curved and straight edge upon the bed line, the curved edge upon the boarding, and the straight edge upon the side of the course, the two edges being in a plane perpendicular to the bed line. The sides of each course being made to agree with every application of the set square, will be what it ought to be. In stone courses, if the stones are truly wrought, the spiral surfaces of the beds will all agree with a set square, and therefore in this case it will be unnecessary to provide one.

APPENDIX,

CONTAINING VARIOUS ARTICLES CONNECTED WITH THE OBLIQUE ARCH.

ARTICLE 1.

To find the solidity of an oblique arch.

RULE.

Multiply the length of the springing line into the length of the arc of the extrados, and this product into the thickness of the arch or breadth of the courses, and the last product will be the solidity in cubic feet, the dimensions being taken in feet.

The length of the arc of the intrados is generally given, being necessary in the construction, and since similar arcs are to one another as their radii, the length of the arc of the extrados will be easily found.

For it is evident that the devolopement of the convex surface is equal to the parallelogram, of which the base is one of the springing lines, and the breadth equal to the length of the arc which has the greater radius, and therefore the solidity may be found by the rule here given.

EXAMPLE.

Let it be required to find the solidity of the arch of which the development is shown in plate 33, the length of the springing line being 37·53 feet, the length of the arc of the intrados 57·32 feet, the radius 25·46, and supposing the thickness of the arch to be 2·5 feet, to find the solidity.

Here $25{\cdot}46+2{\cdot}5=27{\cdot}96$ the radius of the convex surface.

$25{\cdot}46 : 27{\cdot}96 :: 57{\cdot}32 : 62{\cdot}94$ the length of the arc of extrados.

$\therefore\ 37{\cdot}53\times62{\cdot}94\times2{\cdot}5=5905{\cdot}3455$ feet, the solidity required.

This measure, on account of the waste of stone, will not be more than sufficient for the arch stones alone, and if the waste in forming the springers be considered, the quantity cut away would amount to a course of stone equal to the length of the abutment, which ought, therefore, to be added to the solidity of the arch.

ARTICLE 2.

To find the angles for executing an occupation arch with plane joints.

An Occupation Arch is an arch perforating the mound or bank of earth raised to carry a railway, the mound being sustained on each side by a wall which terminates in a battering face.

Let the semi-circle $A\ B\ C$ be a right section of the arch, E being the centre, and $A\ C$ the diameter.

Draw $E\ B$ perpendicular to $A\ C$, and draw $E\ D$, making the angle $B\ E\ D$ equal to the quantity of batter. Divide the arc $A\ B\ C$ into as many equal parts as are arch stones, say nine, at the points 1, 2, 3, &c., and parallel to $A\ C$ draw the lines 1-1′, 2-2′, 3-3′, &c., intersecting $E\ D$ in f, g, h, &c., and meeting the opposite side of the semi-circular arc at the points 1′, 2′, 3′, &c.. Produce $B\ E$ to any convenient point G, and through G draw $P\ R$ parallel to $A\ C$. In $G\ E$ make $G\ k$, $G\ l$, $G\ m$, &c., respectively equal to $E\ f$, $E\ g$, $E\ h$, &c., and through the points $k\ l$, m, &c. parallel to $P\ R$, draw $p\ p'$, $q\ q'$, $r\ r'$, &c. Parallel to $B\ G$ draw $A\ P$, $1\ p$, $2\ q$, $3\ r$, &c.; as also $C\ R$, $1'\ p'$, $2'\ q'$, $3'r'$, &c. Through the points p, q, r, &c., draw the curve $P\ p\ q\ r\ s\ s'\ r'\ q'\ p'\ R$, and from G through the points p, q, r, &c., draw the straight lines $G\ p\ \alpha$, $G\ q\ \beta$, $G\ r\ \gamma$, &c., and the parts $p\ \alpha$, $q\ \beta$, $r\ \gamma$, &c., are the joint lines on the battering face.

In $G\ P$ take $G\ L$ of any convenient length, and draw $L\ H$, $L\ I$, $L\ J$, &c., respectively perpendicular to $G\ s$, $G\ r$, $G\ q$, &c. meeting $G\ s$, $G\ r$, $G\ q$ in H, I, J, &c. Make the angle $G\ L\ M$ equal to the angle $B\ D\ E$, and draw $G\ M$ perpendicular to $G\ L$. In $G\ L$ make $G\ a$, $G\ b$, $G\ c$, &c., respectively equal to $G\ H$, $G\ I$, $G\ J$, &c.; join $a\ M$, $b\ M$, $c\ M$, &c., and the angles $G\ d\ M$, $G\ c\ M$, $G\ b\ M$, &c., or the angles $R\ d\ M$, $R\ c\ M$, $R\ b\ M$, &c., are the dihedral angles made by the planes of the beds and the planes of the faces of the arch at the joints $p\ \alpha$, $q\ \beta$, $r\ \gamma$.

In $L\ P$ make $L\ t$, $L\ u$, $L\ v$, $L\ w$, respectively equal to $L\ H$, $L\ I$, $L\ J$, $L\ K$; from L with the radius $L\ M$ describe the arc $M\ z'\ z\ y\ x$; draw $t\ x$, $u\ y$, $v\ z$, $w\ z'$ perpendicular to $G\ P$; join $L\ x$, $L\ y$, $L\ z$, $L\ z'$ and the angles $P\ L\ z'$, $P\ L\ z$, $P\ L\ y$, $P\ L\ x$, are the angles made by the joint lines of the soffit and the joint lines of the face of the arch, or what has been called the angles of the beds.

The same angles are also to be used for the other half of the arch.

The reader will here recognize the principles of the trihedral, as shown in the introduction (page liv. Prob. xxxviii). The principle of this arch is the same as that of an oblique arch the angle of obliquity being in a vertical instead of a horizontal plan

ARTICLE 3.

To draw the plan and elevation of an occupation arch supposing the ground on one side of it to have a considerable ascent, and the earth before it to be retained by a wall on each side of the approach; each wall being built upon a circular plan and terminated in a regular spiral surface, meeting the face of the arch in its highest point of elevation.

Let *A B Q P* (Fig. 1) be one side of the plan, *A B* the line of the outer face, and *P Q* that of the inner face, *A B* and *P Q* being arcs of circles described from the centre *O*; therefore *A P*, or *B Q* is the thickness of the coping.

Divide the arc *A B* into any convenient number of equal parts at the points 1, 2, 3, &c., and draw the lines 1-1′, 2-2′, 3-3′, &c., radiating to the centre *O*, and meeting the arc *P Q* in the points, 1′, 2′, 3′, &c.

In the elevation (Fig. 2), let the ground line *A′ R′* be parallel to *O B* on the plan. Perpendicular to *A′ R′* draw *A A′*, *B B′*, *C C′* *D D′* intersecting *A′ R′* in *A′ b*, *c*, *d*. Make *b B′* equal to the height of the coping, and make *c C′*, *d D* each equal to the height of the archway. Describe the segment *C′ D′* to the height of the intrados, and *c C′ D′ d* is the elevation of the arch-way or aperture. Divide *b B′* at the points 1, 2, 3, &c., into the same number of parts into which the arc *A B* is divided. Parallel to the ground line *A′ R′* draw 1 *l*, 2 *m*, 3 *n*, &c., and from the points 1, 2, 3, &c., in the arc *A B* draw lines 1 *a*, 2 *b*, 3 *c*, &c., perpendicular to the ground *A′ R′* meeting the lines 1 *l*, 2 *m*, 3 *n*, &c., in the points, *a*, *b*, *c*, *&c.* Also perpendicular to *A′ R′* draw 1′ *l*, 2′ *m*, 3′ *n*, &c. From *A′* through the points *a*, *b*, *c*, &c., draw the curve *A a b c*...*B′*; also from *A′* through the points *l*, *m*, *n*, &c., draw the curve *A l m n* ...*Q*. The curve lines *A′ B′*, *A′ Q′* are the projections of the spiral lines which contain the spiral surface of the coping. The curve line *p q r s*... is the projection of the spiral of the bed of the stone which shows in the cylindric surface of the wall, and thus the two spiral lines represented by *A′ a b c* ..*B′* and *p q r s*... contain that portion of the convex surface of the cylinder which is required to be formed on the edge of the coping.

If the centre of the circular arcs *A B*, *P Q* be in the line *O B* which is the wall line of the face of the arch, the spiral surface terminating the top of the wall will meet the plane of the face of the arch in a straight line parallel to the horizon; but if the centre is not in the

wall line of the face of the arch, the intersection of the spiral surface and the plane of the face of the wall will not be a straight line, but a curve inclined in all points to the horizon. In either case however the method of forming the copings will be the same. It is a matter of minor importance whether the line in which the spiral surface meets, the plane of the face of the arch in a straight or curved line or whether the line of meeting be parallel or inclined to the horizon depending entirely on the taste of the workman or inspector. If the section which would fit upon the face of the arch be inclined, it may be brought to a level by cutting off the projecting point by a plane parallel to the horizon.

Draw the straight line *X Z*, and make *X Z* equal to the length of the arc *A B*, and in *X Z* make *X Y* equal to the length of the arc *P Q*. Draw *X W* perpendicular to *X Z*. Join *W Y* and *W Z*, and *Y W Z* is the angle of the twist.

Article 4.

To form the coping stones for the circular wing walls upon the right and left, see the plan preceding the plate.

Having found the angle of the twist *Y W Z* as shown in the said plate, draw the straight line *a e* of any convenient length, and with the chord *a e* and the radius *O A* or *O B* (Fig. 1, plate 36,) describe the arc *a c e*. Draw *a f* perpendicular to *a e*, and make the angle *a e f* equal to the angle *X Y W* (Fig. 3, plate 36). Draw *e m* perpendicular to *a e*, and *f m* perpendicular to *f e*. Bisect *a e*, *e f*, *f m*, each in the point 2. Draw 2 *c* perpendicular to *a e*, 2 *h* perpendicular to *e f*, and 2 *k* perpendicular to *f m*. Make 2 *h*, 2 *k*, each equal to 2 *c*, and describe the arcs *e h f*, *f k m*.

Then the bed of the stone being formed to the twist, or spiral surface, as in the oblique arch, the narrow face, which is the edge of the stone, will be formed to the cylindric surface of the wall by the arch square No 3, observing that as the face of the wall corresponding to the arc *A B* (Fig. 1, plate 36,) is convex, the curved limb of the inner edge of the arch square must be concave.

The bed and upper surface of the coping for the left hand cylindric wall are right hand spiral surfaces, and the bed and upper surface of the coping of the right hand cylindric wall are left hand spiral surfaces; for the plan of the wall on the left hand is the same as the plan of a right hand spiral stair, and the plan of the wall on the right hand the same as a left hand spiral stair.

ARTICLE 5.

To construct an oblique Arch in a battering wall.

Let *C M K* (Fig. 1) be the angle of obliquity, *F B C* the right section of the arch, and *V W X* the complement of the angle of the batter of the wall or the dihedral angle, which the face of the wall makes with the level or horizontal plane of the base. Prolong *C F* and *M K* to meet each other in *A*.

Then in a trihedral are given the angle F A K of the adjacent face, and the dihedral angle adjacent, to find the angle of the opposite face and the angle of the oblique face.

In A F take any convenient point G, and draw G I intersecting A K perpendicularly in H. "In A F make G O equal to G H, and make the angle G O E equal to the given dihedral angle v w x. Draw perpendicular to A G, and through E draw A J, and the angle F A J is the angle of the opposite face."

"*From A, with the distance A E, cut H I in I, and through I draw A L, and the angle K A L is the angle of the oblique face,*" (as in Prob. xxiii. introduction).

Prolong *C F* to *P*, and make *F P* equal to the length of the arc *F B C*. Divide the arc *F B C* and the straight line *F P* each into an equal number of equal parts at the points, 1, 2, 3, &c. From the points 1, 2, 3, &c. in the arc *F B C* draw 1 *m*, 2 *n*, 3 *o*, &c. parallel to *A J*, meeting *E C* in the points *m*, *n*, *o*, &c. Parallel to *C M* draw *F K*, *m p*, *n q*, *o r*, &c, meeting *K M* in *K*, *p*, *q*, *r*, &c. Parallel *A L*, draw *p s*, *q t*, *r u*, &c , and make *p s*, *q t*, *r u*, &c. respectively equal to *m* 1, *n* 2, *o* 3, &c., and through the points *s*, *t*, *u*, &c., draw the curve *K N M*, which is the section of the cylinder or the under edge of the elevation of the oblique face of the arch.

Through the points 1, 2, 3, &c. in the straight line *E P* parallel to *C M* draw 1 p, 2 q, 3 r, &c. Make 1 p, 2 q, 3 r, &c. respectively equal to *m p*, *n q*, *o r*, &c.; through the points *K*, p, q, r, &c. draw a curve and the curve *K* p q r... *Q* shall be the developement of the oblique section *K N M*.

Draw the straight line *K Q*, and having divided it into as many equal parts as the courses are in number, and having the length of the springing line which is parallel to *C M* given ; the developement of the intrados and all the necessary templets and angles for working the stones may be found by proceeding in the usual manner as when the two faces are vertical and in parallel planes.

But in the construction of an oblique arch in a battering wall the curve line of the developement of the arch differs more from its line of subtense *K Q* than when the planes of the elevation are vertical and parallel; and more particularly so when in the oblique arch with vertical faces, the less part of the arc of the right section is of the entire circle. Oblique arches which have battering faces are therefore not so strong as those which have their faces in parallel planes.

ARTICLE 6.

To describe a semi-oval from five centres, so that the curve may be a near approximation to a semi-ellipse of given dimensions.

Let $A\ B$ (Fig. 1) be equal to the axis major. Bisect $A\ B$ in C by the pendicular $D\ E$, and make $C\ D$ equal to the semi-axis minor.

Draw two straight lines $o\ t, o\ s$, (No. 2) forming any convenient angle. In $o\ t$ and $o\ s$ make $o\ p$, $o\ q$ each equal to $C\ D$, and $o\ r$, $o\ s$, each equal to $C\ A$ or $C\ B$. Join $q\ r$ and draw $s\ t$ and $p\ u$ parallel to $q\ r$. Make $D\ E$ equal to $o\ t$, $A\ F$ and $B\ F'$ each equal to $o\ u$.

Divide $E\ C$ into two equal parts in h, and divide $F\ C$, $F'\ C$ each into three equal parts, making $F\ k$, $F'\ m$ each equal to one part, and $C\ k$, $C\ m$ will be each equal to two parts. From E through k and m draw $E\ c$, $E\ d$; and from h through F and F' draw $h\ b$, $h\ e$ intersecting $E\ c$, in 2, and draw $h\ e$, intersecting $E\ d$ in 4. From F, which is the first centre, describe the arc $A\ b$; from 2 describe the arc $b\ c$; from E describe the arc $c\ d$; from 4 describe the arc $d\ e$; and from F' describe the arc $e\ B$.

ARTICLE 7.

To describe a semi-oval from seven centres, so that the curve may be a very near approximation to the figure of a semi-ellipse.

Let $A\ B$ be equal to the axis major. Bisect $A\ B$ in C by the perpendicular $D\ E$, and make $C\ D$ equal to the semi-axis minor.

Find the points F, F' by finding the radius of curvature at the points A and B, and find the point E by the radius of curvature at D. Thus let $C\ D$ be 6 feet, and CA or $C\ B$ be 12 feet; then by proportion

$$C\ B \text{ or } C\ A : C\ D :: C\ D : A\ F = \frac{C\ D^2}{C\ A} = \frac{6^2}{12} = \frac{36}{12} = 3$$

$$\text{and } C\ D : C\ A :: C\ A : D\ E = \frac{C\ A^2}{C\ D} = \frac{12^2}{6} = \frac{144}{6} = 24$$

Make $D\ E$ equal to 24 feet, and in $A\ B$ make $A\ F$, $B\ F'$ each equal to 3 feet. Divide $E\ C$ into three equal parts at i, h, and divide $F\ C$ as also $F'\ C$ into three parts each in the proportion of 1, 2, 3; so that by dividing $F\ C$ into six equal parts make $F\ k$ equal to 1, $k\ l$ equal to 2, and $l\ C$ equal to 3, as also make $F'\ m$ equal to 1, $m\ n$ equal to 2, and $n\ c$ equal to 3. From E through l and n draw $E\ d$, $E\ e$; from i through k draw $i\ c$, intersecting $E\ d$ in 3, and through m draw $i\ f$, intersecting $E\ e$ in 5; and from h through F draw $h\ b$, intersecting $3\ c$ in 2, and through F' draw $h\ g$, intersecting $5\ f$ in 6, and the seven centres are F, 2, 3, E, 5, 6, F'. From F with the radius $F\ A$ describe the arc $A\ b$; from 2 with the radius $2\ c$ describe the arc $b\ c$; from 3 with the radius $3\ c$ describe the arc $c\ d$; from E with the radius $E\ d$ describe the arc $d\ e$, and so on; then the curves described by the circular arcs will be a very near approximation to the semi-ellipse.

In the same manner may a semi-oval be described from nine, eleven, &c. centres, by always dividing $F\ C$, $F'\ C$ each into as many parts as there are equal parts in $C\ E$ in the proportion of the arithmetical progression 1, 2, 3, &c., the first term and the common difference being unity.

Observations.

The method here shown of describing a semi-oval which shall represent a semi-ellipse is not so near an approximation as that shown page xiii (Introduction) figure 2. This construction is taken from a French publication, 1792*; so that these methods are not new. The author of this work has added the method of finding the axis which is not shown in the said publication, and indeed it is not there intended that the semi-oval shall be a semi-ellipse, but a figure of much greater capacity being adapted to the construction of the arches of bridges, in order to admit of the passage of a greater quantity of water. For instead of finding the axis $A\,B$, $C\,D$, fix upon the radius $A\,F$ or $B\,F'$ at pleasure; and take $C\,E$ equal to three times $C\,F$ or three times $C\,F'$, Divide $C\,E$ into three equal parts, and $C\,F$ or $C\,F'$ into three parts in the proportion of the numbers 1, 2, 3. Join the points of division by the lines $k\,F$, $i\,k$, $E\,l$, and the points F, 2, 3, E, 5, 6, F, are the centres. So that if an elliptic curve be given or described by any means, the centres for drawing the joints perpendicular to the curve may be easily determined.

* Traité Elémentaire de la Coupe des Pierres ou Art du Trait par Mr. Simonin Professeur de Mathematiques, Paris. MDCCLXXXXII.

	40 Deg.		41 Deg.		42 Deg.		43 Deg.		44 Deg.		
M.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	6135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	49 Deg.		48 Deg.		47 Deg.		46 Deg.		45 Deg.		

face of the work they are represented by lines. In laying a course of stones, the joints between every two stones is called the heading joint, or simply the joint, and in laying two courses the thin sheet of mortar is called the bed joint, and the line which appears in the face of the work is called the bed line.

In the oblique arch the bed lines which are the visible edges of the bed joints show a series of spiral lines upon the cylindric surface of the intrados, and the developement of these lines becomes parallel straight lines, the heading joints are not always arranged in spiral lines, as it is not always convenient to procure stones of sufficient length; but when this can be done the intrados will have a much more elegant appearance.

Intrados of an oblique arch is the underside or concave cylindric surface exhibiting all the joint lines. The cylindric surface is contained by four sides of which the two horizontal sides are the springing lines, and the remaining two are the quoins of the arch; so that the figure of the developement is a quadrilateral contained by two parallel straight lines, and two identical curve lines of contrary flexure, such that straight lines drawn from the ends of the curves will be bisected by the curves. In the intermediate surface of the intrados, the bed lines exhibit a series of equidistant spirals, which, when developed upon a plane surface, become as many equidistant parallel straight lines; and when the joints are arranged in spirals, they become in their developement also a series of equidistant parallel straight lines all equal to one another, being terminated by the springers which are parallel.

Line of subtense, in the developement of the cylindric surface cut by the plane of the oblique face of the arch, is a straight line joining the ends of each curve which terminate the ends of the developement. The lines of subtense determine the position both of the bed and joint lines.

Oblique arch is an arch of which the springing lines are not perpendicular to the faces. Oblique arches are of two kinds, the one having their joints in plane surfaces, and the other in spiral surfaces.

Quoin of an oblique arch is the arris which separates the intrados from the plane of elevation, or, the curve line in which the cylindric surface and the oblique face meet each other. The developement of the cylindric surface which contains the quoin is a curve of contrary flexure, in the form of the letter *S*, but flatter. The quoin is divided by the bed lines into parts which are very nearly equal to one another. The figure of the quoin is a semi-ellipse or a segment less than a semi-ellipse when the right section is a segment less than a semi-circle.

Quoin stones of an oblique arch are those stones which contain the quoin, and therefore show a part of the elevation and a part of the intrados.

Radius of curvature of the section of a cylinder is the radius of a circle which shall coincide at least for a considerable distance with the curve which is formed by cutting the cylinder by a plane.

Ring Stones are those stones which form the quoin of the arch by the meeting of the cylindric surface of the intrados, and the plane surface of the face. In the elevation of an oblique arch the bed lines which are sections of spiral surfaces, are curves of which the radius of curvature is least at each springing, but continually greater upwards, till at the summit it becomes infinitely long; or, which is the same thing, the deflection of the curve line joints from straight lines is continually less and nothing at the summit. These curved joints have in general so little deflection in their length that the greatest distance between the straight line and the curve, if not imperceptible, could not well be measured except under a magnifying power; they are, therefore, generally represented by straight lines which meet each other in a certain point in the minor axis.

Right section of an oblique arch is a section perpendicular to the axis of the cylinder. The length of the arc of this section gives the breadth of the parallelogram which coincides with the outline of the arch, the distance between the springing lines is equal to the length of the arc of the right section. The right section intersects the bed lines at oblique angles, so as to make the acute angles equal to the complements of the angles of the intrados.

Springers or imposts are those serrated stones which in profile form a series of indentations and risings something like the teeth of a joiner's saw. The sides of these teeth from the springing line is a portion of the intrados, the cylindric surface of which when developed becomes a series of identical right angled triangles, of which their longest sides make the entire length of the springing line. Of the other two sides which contain the right angle of each of these triangles, the one is equal to the breadth of the courses, and the remaining one equal to half the length of a stone, supposing the joint between every two stones in every course to fall upon the middle of a stone in each of the adjacent courses. Besides the cylindric surface now explained, every springer exhibits two spiral surfaces, the one of which is called the back, and the other the head. The bed of a course of springers is in one horizontal plane which is generally below the springing line. See plate 25, figures 2 and 3.

Square section of an oblique arch is the same as right section.

Vossoirs are the arch stones of any arch. In the oblique arch all the stones excepting the abutments or piers are arch stones; those in the face are the quoin stones and the remaining ones are the common arch stones; each of the common arch stones must be wrought on five sides, four of these sides are spiral surfaces; two belong to the beds and two to the ends; the remaining side is the face which is a portion of the cylindric surface.

TABLE

OF

NATURAL SINES,

FOR THE USE

OF THE

PRECEDING TREATISE ON THE OBLIQUE ARCH.

SEE SECTION VI.—INTRODUCTION.

TABLE OF NATURAL SINES.

	0 Deg.		1 Deg.		2 Deg.		3 Deg.		4 Deg.		
M.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
0	0	100000	1745	99985	3490	99939	5234	99863	6976	99756	60
1	29	100000	1774	99984	3519	99938	5263	99861	7005	99754	59
2	58	100000	1803	99984	3548	99937	5292	99860	7034	99752	58
3	87	100000	1832	99983	3577	99936	5321	99858	7063	99750	57
4	116	100000	1862	99983	3606	99935	5350	99857	7092	99748	56
5	145	100000	1891	99982	3635	99934	5379	99855	7121	99746	55
6	175	100000	1920	99982	3664	99933	5408	99854	7150	99744	54
7	204	100000	1949	99981	3693	99932	5437	99852	7179	99742	53
8	233	100000	1978	99980	3723	99931	5466	99851	7208	99740	52
9	262	100000	2007	99980	3752	99930	5495	99849	7237	99738	51
10	291	100000	2036	99979	3781	99929	5524	99847	7266	99736	50
11	320	99999	2065	99979	3810	99927	5553	99846	7295	99734	49
12	349	99999	2094	99978	3839	99926	5582	99844	7324	99731	48
13	378	99999	2123	99977	3868	99925	5611	99842	7353	99729	47
14	407	99999	2152	99977	3897	99924	5640	99841	7382	99727	46
15	436	99999	2181	99976	3926	99923	5669	99839	7411	99725	45
16	465	99999	2211	99976	3955	99922	5698	99838	7440	99723	44
17	495	99999	2240	99975	3984	99921	5727	99836	7469	99721	43
18	524	99999	2269	99974	4013	99919	5756	99834	7498	99719	42
19	553	99998	2298	99974	4042	99918	5785	99833	7527	99716	41
20	582	99998	2327	99973	4071	99917	5814	99831	7556	99714	40
21	611	99998	2356	99972	4100	99916	5844	99829	7585	99712	39
22	640	99998	2385	99972	4129	99915	5873	99827	7614	99710	38
23	669	99998	2414	99971	4159	99913	5902	99826	7643	99708	37
24	698	99998	2443	99970	4188	99912	5931	99824	7672	99705	36
25	727	99997	2472	99969	4217	99911	5960	99822	7701	99703	35
26	756	99997	2501	99969	4246	99910	5989	99821	7730	99701	34
27	785	99997	2530	99968	4275	99909	6018	99819	7759	99699	33
28	814	99967	2560	99967	4304	99907	6047	99817	7788	99696	32
29	844	99996	2589	99966	4333	99906	6076	99815	7817	99694	31
30	873	99996	2618	99966	4362	99905	6105	99813	7846	99692	30
31	902	99996	2647	99965	4391	99904	6134	99812	7875	99689	29
32	931	99996	2676	99964	4420	99902	6163	99810	7904	99687	28
33	960	99995	2705	99963	4449	99901	6192	99808	7933	99685	27
34	989	99995	2734	99963	4478	99900	6221	99806	7962	99683	26
35	1018	99995	2763	99962	4507	99898	6250	99804	7991	99680	25
36	1047	99995	2792	99961	4536	99897	6279	99803	8020	99678	24
37	1076	99994	2821	99960	4565	99896	6308	99801	8049	99676	23
38	1105	99994	2850	99959	4594	99894	6337	99799	8078	99673	22
39	1134	99994	2879	69959	4623	99893	6366	99797	8107	99671	21
40	1164	99993	2908	99958	4653	99892	6395	99795	8136	99668	20
41	1193	99993	2938	99957	4682	99890	6424	99793	8165	99666	19
42	1222	99993	2967	99956	4711	99889	6453	99792	8194	99664	18
43	1251	99992	2996	99955	4740	99888	6482	99790	8223	99661	17
44	1280	99992	3025	99954	4769	99886	6511	99788	8252	99659	16
45	1309	99991	3054	99953	4798	99885	6540	99786	8281	99657	15
46	1338	99991	3083	99952	4827	99883	6569	99784	8310	99654	14
47	1367	99991	3112	99952	4856	99882	6598	99782	8339	99652	13
48	1396	99990	3141	99951	4885	99881	6627	99780	8368	99649	12
49	1425	99990	3170	99950	4914	99879	6656	99778	8397	99647	11
50	1454	99989	3199	99949	4943	99878	6685	99776	8426	99644	10
51	1483	99989	3228	99948	4972	99876	6714	99774	8455	99642	9
52	1513	99989	3257	99947	5001	99875	6743	99772	8484	99639	8
53	1542	99988	3286	99946	5030	99873	6773	99770	8513	99637	7
54	1571	99988	3316	99945	5059	99872	6802	99768	8542	99635	6
55	1600	99987	3345	99944	5088	99870	6831	99766	8571	99632	5
56	1629	99987	3374	99943	5117	99869	6860	99764	8600	99630	4
57	1658	99986	3403	99942	5146	99867	6889	99762	8629	99627	3
58	1687	99986	3432	99941	5175	99866	6918	99760	8658	99625	2
59	1716	99985	4461	99940	5205	99864	6947	99758	8687	99622	1
60	1745	99985	3490	99939	5234	99863	6976	99756	8716	99619	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	89 Deg.		88 Deg.		87 Deg.		86 Deg.		85 Deg.		

eg.	6 Deg.		7 Deg.		8 Deg.		9 Deg.		
N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
99619	10453	99452	12187	99255	13917	99027	15643	98769	60
99617	10482	99449	12216	99251	13946	99023	15672	98764	59
99614	10511	99446	12245	99248	13975	99019	15701	98760	58
99612	10540	99443	12274	99244	14004	99015	15730	98755	57
99609	10569	99440	12302	99240	14033	99011	15758	98751	56
99607	10597	99437	12331	99237	14061	99006	15787	98746	55
99604	10626	99434	12360	99233	14090	99002	15816	98741	54
99602	10655	99431	12389	99230	14119	98998	15845	98737	53
99599	10684	99428	12418	99226	14148	98994	15873	98732	52
99596	10713	99424	12447	99222	14177	98990	15902	98728	51
99594	10742	99421	12476	99219	14205	98986	15931	98723	50
99591	10771	99418	12504	99215	14234	98982	15959	98718	49
99588	10800	99415	12533	99211	14263	98978	15988	98714	48
99586	10829	99412	12562	99208	14292	98973	16017	98709	47
99583	10858	99409	12591	99204	14320	98969	16046	98704	46
99580	10887	99406	12620	99200	14349	98965	16074	98700	45
99578	10916	99402	12649	99197	14378	98961	16103	98695	44
99575	10945	99399	12678	99193	14407	98957	16132	98690	43
99572	10973	99396	12706	99189	14436	98953	16160	98686	42
99570	11002	99393	12735	99186	14464	98948	16189	98681	41
99567	11031	99390	12764	99182	14493	98944	16218	98676	40
99564	11060	99386	12793	99178	14522	98940	16246	98671	39
99562	11089	99383	12822	99175	14551	98936	16275	98667	38
99559	11118	99380	12851	99171	14580	98931	16304	98662	37
99556	11147	99377	12880	99167	14608	98927	16333	98657	36
99553	11176	99374	12908	99163	14637	98923	16361	98652	35
99551	11205	99370	12937	99160	14666	98919	16390	98648	34
99548	11234	99367	12966	99156	14695	98914	16419	98643	33
99545	11263	99364	12995	99152	14723	98910	16447	98638	32
99542	11291	99360	13024	99148	14752	98906	16476	98633	31
99540	11320	99357	13053	99144	14781	98902	16505	98629	30
99537	11349	99354	13081	99141	14810	98897	16533	98624	29
99534	11378	99351	13110	99137	14838	98893	16562	98619	28
99531	11407	99347	13139	99133	14867	98889	16591	98614	27
99528	11436	99344	13168	99129	14896	98884	16620	98609	26
99526	11465	99341	13197	99125	14925	98880	16648	98604	25
99523	11494	99337	13226	99122	14954	98876	16677	98600	24
99520	11523	99334	13254	99118	14982	98871	16706	98595	23
99517	11552	99331	13283	99114	15011	98867	16734	98590	22
99514	11580	99327	13312	99110	15040	98863	16763	98585	21
99511	11609	99324	13341	99106	15069	98858	16792	98580	20
99508	11638	99320	13370	99102	15097	98854	16820	98575	19
99506	11667	99317	13399	99098	15126	98849	16849	98570	18
99503	11696	99314	13427	99094	15155	98845	16878	98565	17
99500	11725	99310	13456	99091	15184	98841	16906	98561	16
99497	11754	99307	13485	99087	15212	98836	16935	98556	15
99494	11783	99303	13514	99083	15241	98832	16964	98551	14
99491	11812	99300	13543	99079	15270	98827	16992	98546	13
99488	11840	99297	13572	99075	15299	98823	17021	98541	12
99485	11869	99293	13600	99071	15327	98818	17050	98536	11
99482	11898	99290	13629	99067	15356	98814	17078	98531	10
99479	11927	99286	13658	99063	15385	98809	17107	98526	9
99476	11956	99283	13687	99059	15414	98805	17136	98521	8
99473	11985	99279	13716	99055	15442	98800	17164	98516	7
99470	12014	99276	13744	99051	15471	98796	17193	98511	6
99467	12043	99272	13773	99047	15500	98791	17222	98506	5
99464	12071	99269	13802	99043	15529	98787	17250	98501	4
39461	12100	99265	13831	99039	15557	98782	17279	98496	3
99458	12129	99262	13860	99035	15586	98778	17308	98491	2
99455	12158	99258	13889	99031	15615	98773	17336	98486	1
99452	12187	99255	13917	99027	15643	98769	17365	98481	0
N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
eg.	83 Deg.		82 Deg.		81 Deg.		80 Deg.		

	10 Deg.		11 Deg.		12 Deg.		13 Deg.		14 Deg.		
M.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18133	98341	19851	98010	21559	97648	23260	97257	24954	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
60	19081	98163	20791	97815	22495	97437	24192	97030	25882	96593	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	79 Deg.		78 Deg.		77 Deg.		76 Deg.		75 Deg.		

Deg.	16 Deg.		17 Deg.		18 Deg.		19 Deg.		
N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
96593	27564	96126	29237	95630	30902	95106	32557	94552	60
96585	27592	96118	29265	95622	30929	95097	32584	94542	59
96578	27620	96110	29293	95613	30957	95088	32612	94533	58
96570	27648	96102	29321	95605	30985	95079	32639	94523	57
96562	27676	96094	29348	95596	31012	95070	32667	94514	56
96555	27704	96086	29376	95588	31040	95061	32694	94504	55
96547	27731	96078	29404	95579	31068	95052	32722	94495	54
96540	27759	96070	29432	95571	31095	95043	32749	94485	53
96532	27787	96062	29460	95562	31123	95033	32777	94476	52
96524	27815	96054	29487	95554	31151	95024	32804	94466	51
96517	27843	96046	29515	95545	31178	95015	32832	94457	50
96509	27871	96037	29543	95536	31206	95006	32859	94447	49
96502	27899	96029	29571	95528	31233	94997	32887	94438	48
96494	27927	96021	29599	95519	31261	94988	32914	94428	47
96486	27955	96013	29626	95511	31289	94979	32942	94418	46
96479	27983	96005	29654	95502	31316	94970	32969	34409	45
96471	28011	95997	29682	95493	31344	94961	32997	94399	44
96463	28039	95989	29710	95485	31372	94952	33024	94390	43
96456	28067	95981	29737	95476	31399	94943	33051	94380	42
96448	28095	95972	29765	95467	31427	94933	33079	94370	41
96440	28123	95964	29793	95459	31454	94924	33106	94361	40
96433	28150	95956	29821	95450	31482	94915	33134	94351	39
96425	28178	95948	29849	95441	31510	94906	33161	94342	38
96417	28206	95940	29876	95433	31537	94897	33189	94332	37
96410	28234	95931	29904	95424	31565	94888	33216	94322	36
96402	28262	95923	29932	95415	31593	94878	33244	94313	35
96394	28290	95915	29960	95407	31620	94869	33271	94303	34
96386	28318	95907	29987	95398	31648	94860	33298	94293	33
96379	28346	95898	30015	95389	31675	94851	33326	94284	32
96371	28374	95890	30043	95380	31703	94842	33353	94274	31
96363	28402	95882	30071	95372	31730	94832	33381	94264	30
96355	28429	95874	30098	95363	31758	94823	33408	94254	29
96347	28457	95865	30126	95354	31786	94814	33436	94245	28
96340	28485	95857	30154	95345	31813	94805	33463	94235	27
96332	28513	95849	30182	95337	31841	94795	33490	94225	26
96324	28541	95841	30209	95328	31868	94786	33518	94215	25
96316	28569	95832	30237	95319	31896	94777	33545	94206	24
96308	28597	95824	30265	95310	31923	94768	33573	94196	23
96301	28625	95816	30292	95301	31951	94758	33600	94186	22
96293	28652	95807	30320	95293	31979	94749	33627	94176	21
96285	28680	95799	30348	95284	32006	94740	33655	94167	20
96277	28708	95791	30376	95275	32034	94730	33682	94157	19
96269	28736	95782	30403	95266	32061	94721	33710	94147	18
96261	28764	95774	30431	95257	32089	94712	33737	94137	17
96253	28792	95766	30459	95248	32116	94702	33764	94127	16
96246	28820	95757	30486	95240	32144	94693	33792	94118	15
96238	28847	95749	30514	95231	32171	94684	33819	94108	14
96230	28875	95740	30542	95222	32199	94674	33846	94098	13
96222	28903	95732	30570	95213	32227	94665	33874	94088	12
96214	28931	95724	30597	95204	32254	94656	33901	94078	11
96206	28959	95715	30625	95195	32282	94646	33929	94068	10
96198	28987	95707	30653	95186	32309	94637	33956	94058	9
96190	29015	95698	30680	95177	32337	94627	33983	94049	8
96182	29042	95690	30708	95168	32364	94618	34011	94039	7
96174	29070	95681	30736	95159	32392	94609	34038	94029	6
96166	29098	95673	30763	95150	32419	94599	34065	94019	5
96158	29126	95664	30791	95142	32447	94590	34093	94009	4
96150	29154	95656	30819	95133	32474	94580	34120	93999	3
96142	29182	95647	30846	95124	32502	94571	34147	93989	2
96134	29209	95639	30874	95115	32529	94561	34175	93979	1
96126	29237	95630	30902	95106	32557	94552	34202	93969	0
N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
Deg.	73 Deg.		72 Deg.		71 Deg.		70 Deg.		

	25 Deg.		26 Deg.		27 Deg.		28 Deg.		29 Deg.		
M.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N sin.	M.
	64 Deg.		63 Deg.		62 Deg.		61 Deg.		60 Deg.		

	30 Deg.		31 Deg.		32 Deg.		33 Deg.		34 Deg.		
M.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	59 Deg.		58 Deg.		57 Deg.		56 Deg.		55 Deg.		

M.	35 Deg. N. sin.	35 Deg. N. cos.	36 Deg. N. sin.	36 Deg. N. cos.	37 Deg. N. sin.	37 Deg. N. cos.	38 Deg. N. sin.	38 Deg. N. cos.	39 Deg. N. sin.	39 Deg. N. cos.	
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78693	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	80713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59762	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79016	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76611	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	54 Deg.		53 Deg.		52 Deg.		51 Deg.		50 Deg.		

	40 Deg.		41 Deg.		42 Deg.		43 Deg.		44 Deg.		
M.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	6135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	49 Deg.		48 Deg.		47 Deg.		46 Deg.		45 Deg.		

PLATE. 7.

Introduction

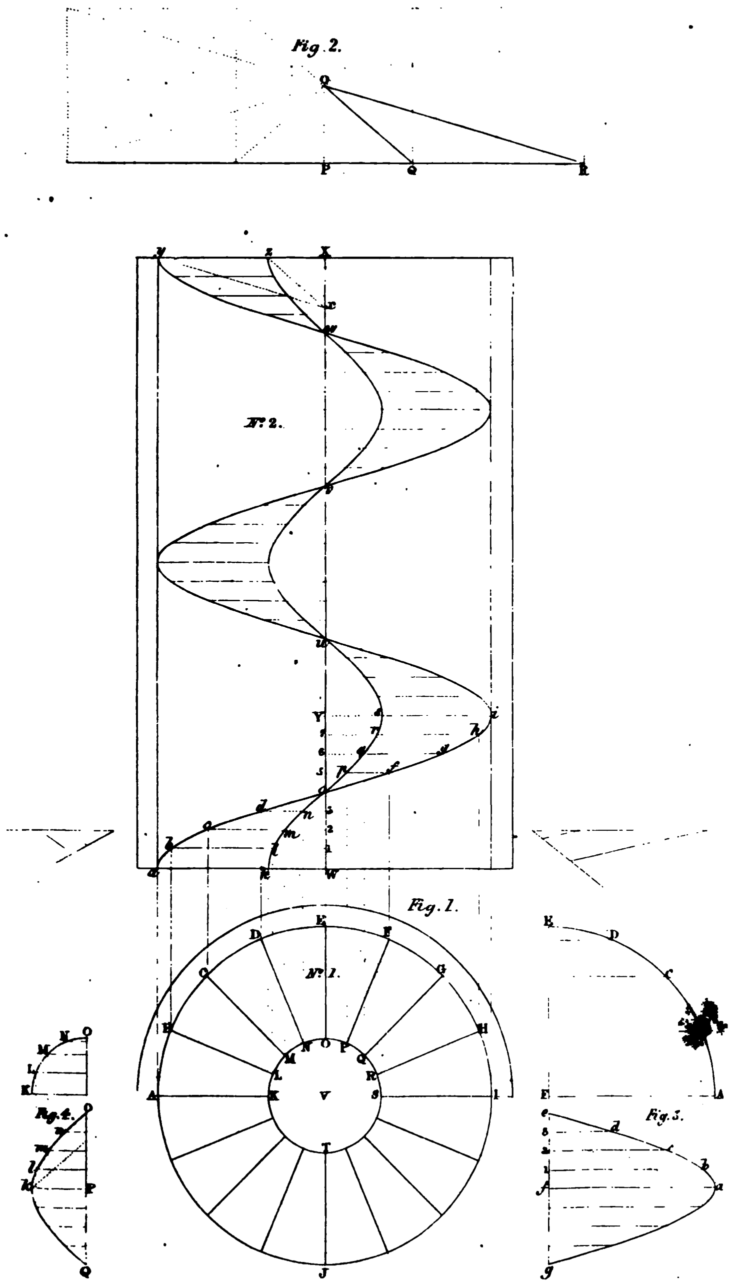


PLATE.8.

Introduction

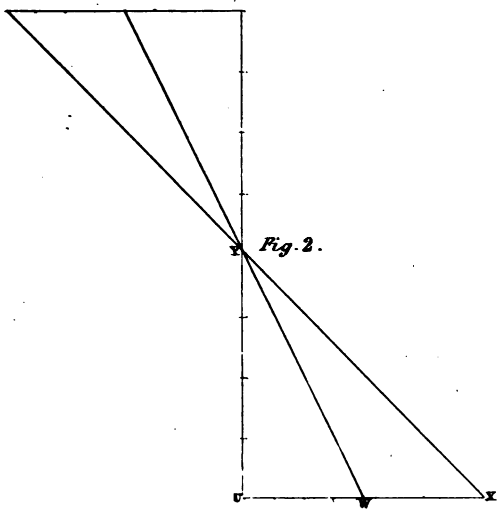


Fig. 2.

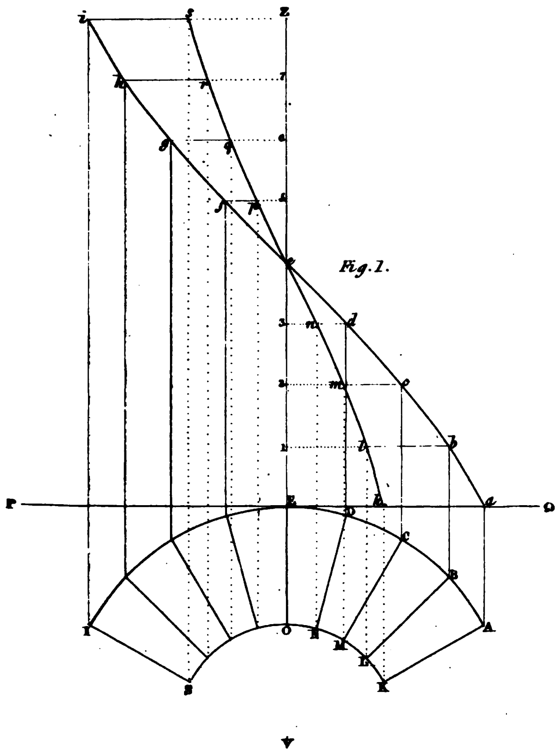


Fig. 1.

PLATE. 9.

Introduction

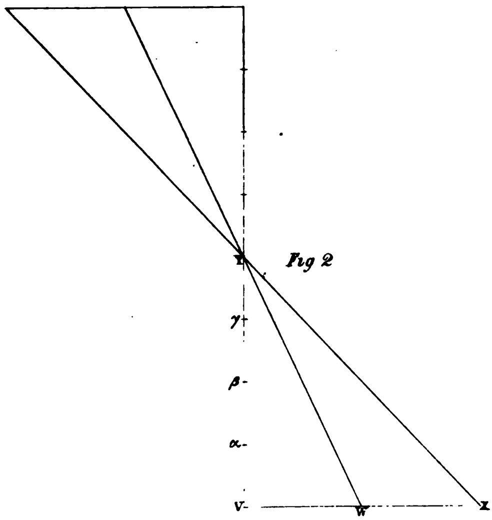


Fig 2

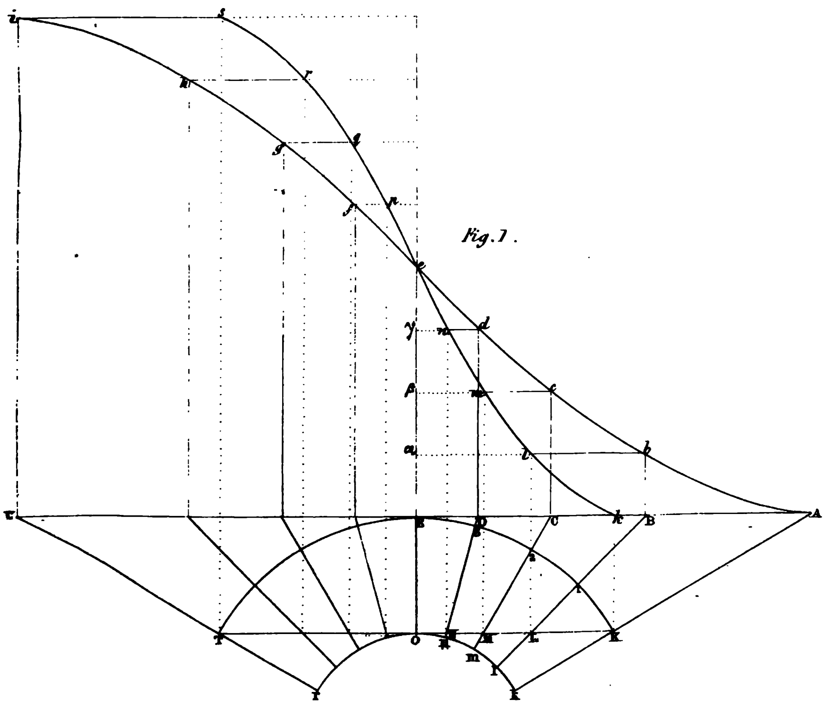


Fig. 7.

PLATE. 4.

Introduction

Fig. 1.

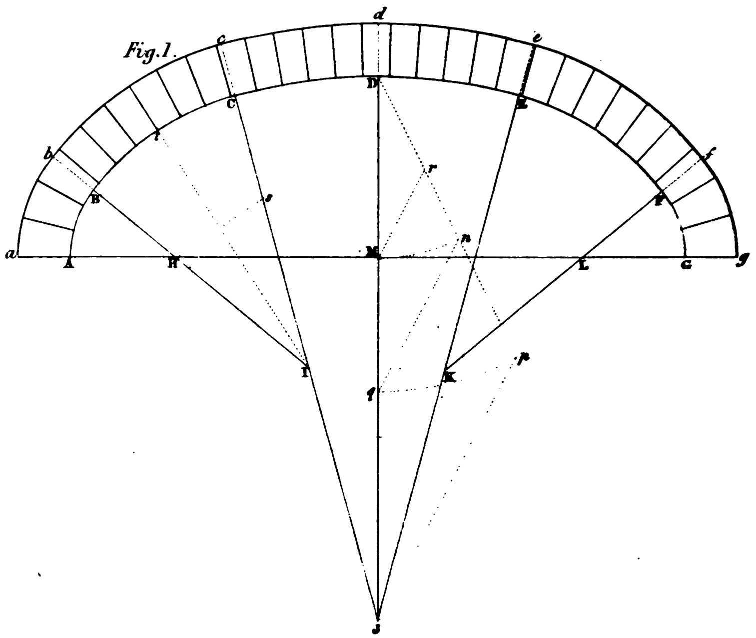


Fig. 2.

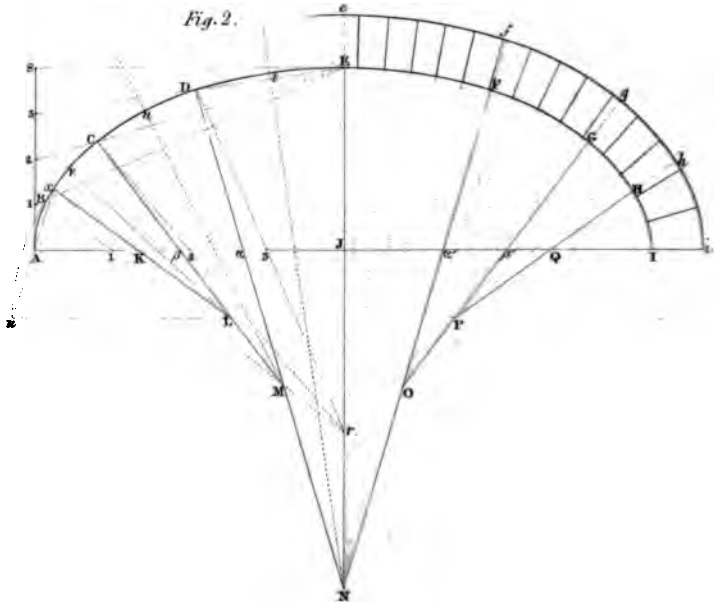


PLATE II.

Introduction

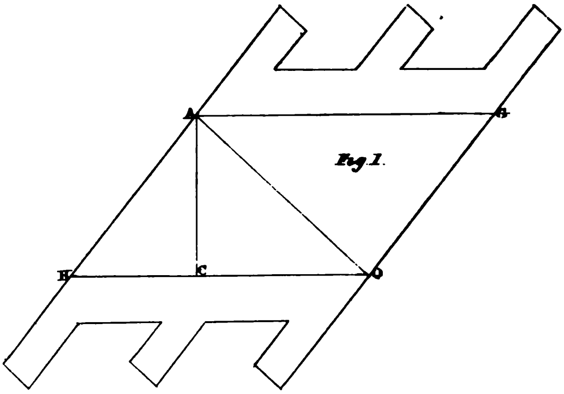


Fig. 1.

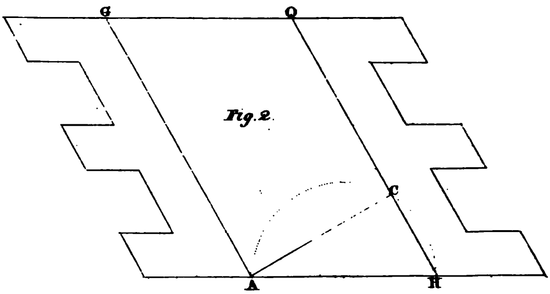


Fig. 2.

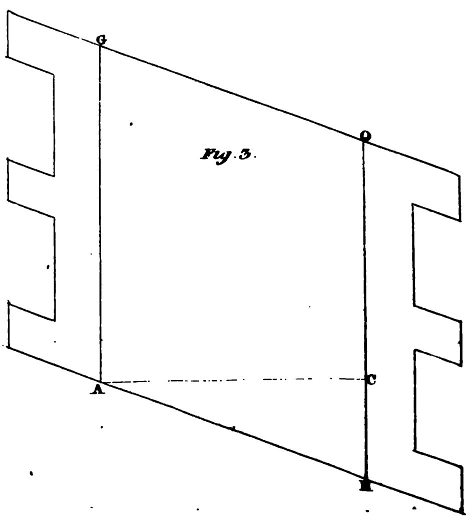
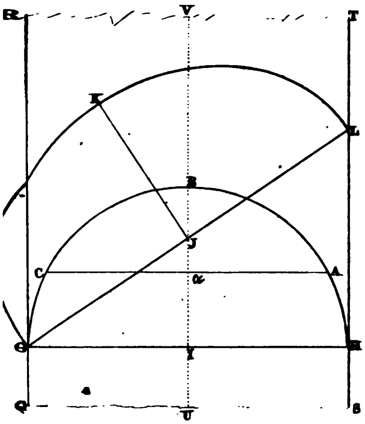


Fig. 3.

PLATE 12.

Introduction



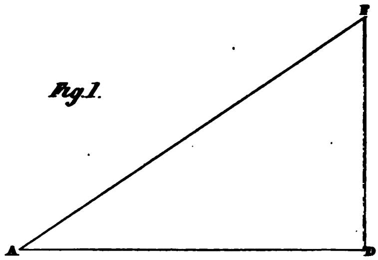
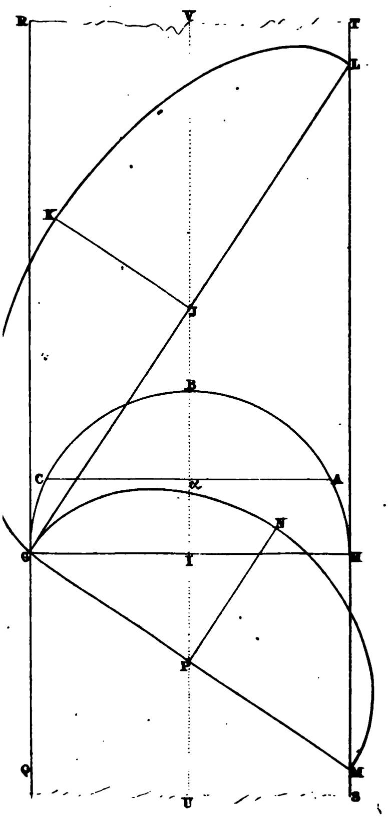


Fig. 1.



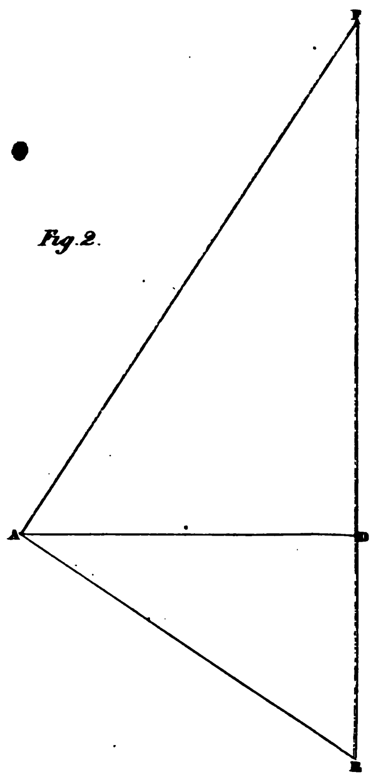


Fig. 2.

PLATE 13.

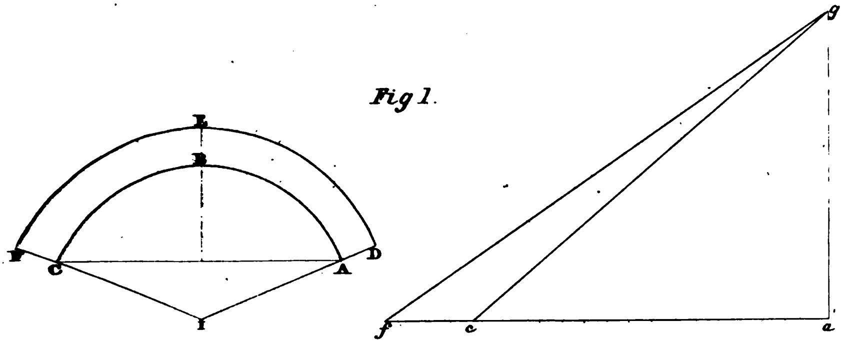


Fig 1.

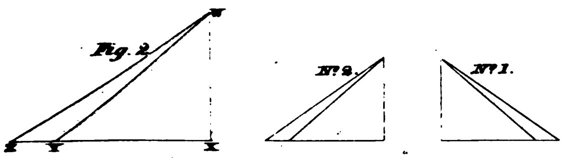


Fig. 2.

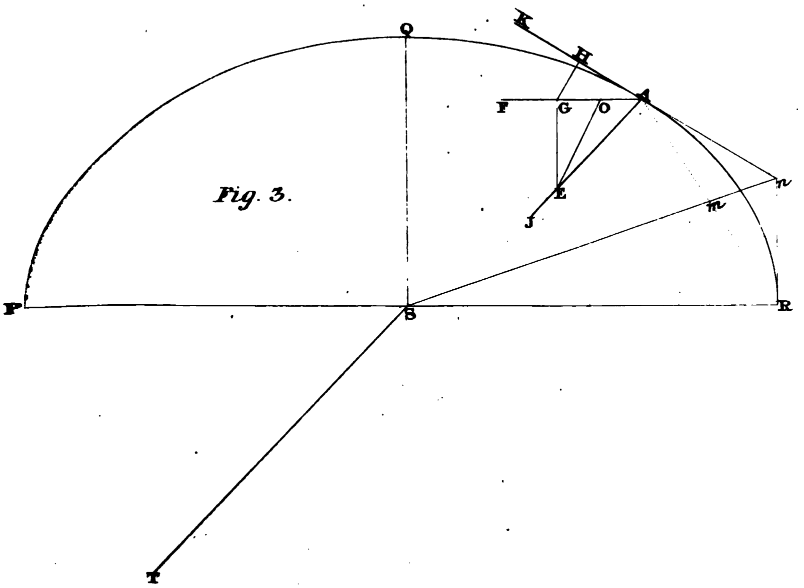


Fig. 3.

PLATE 14.

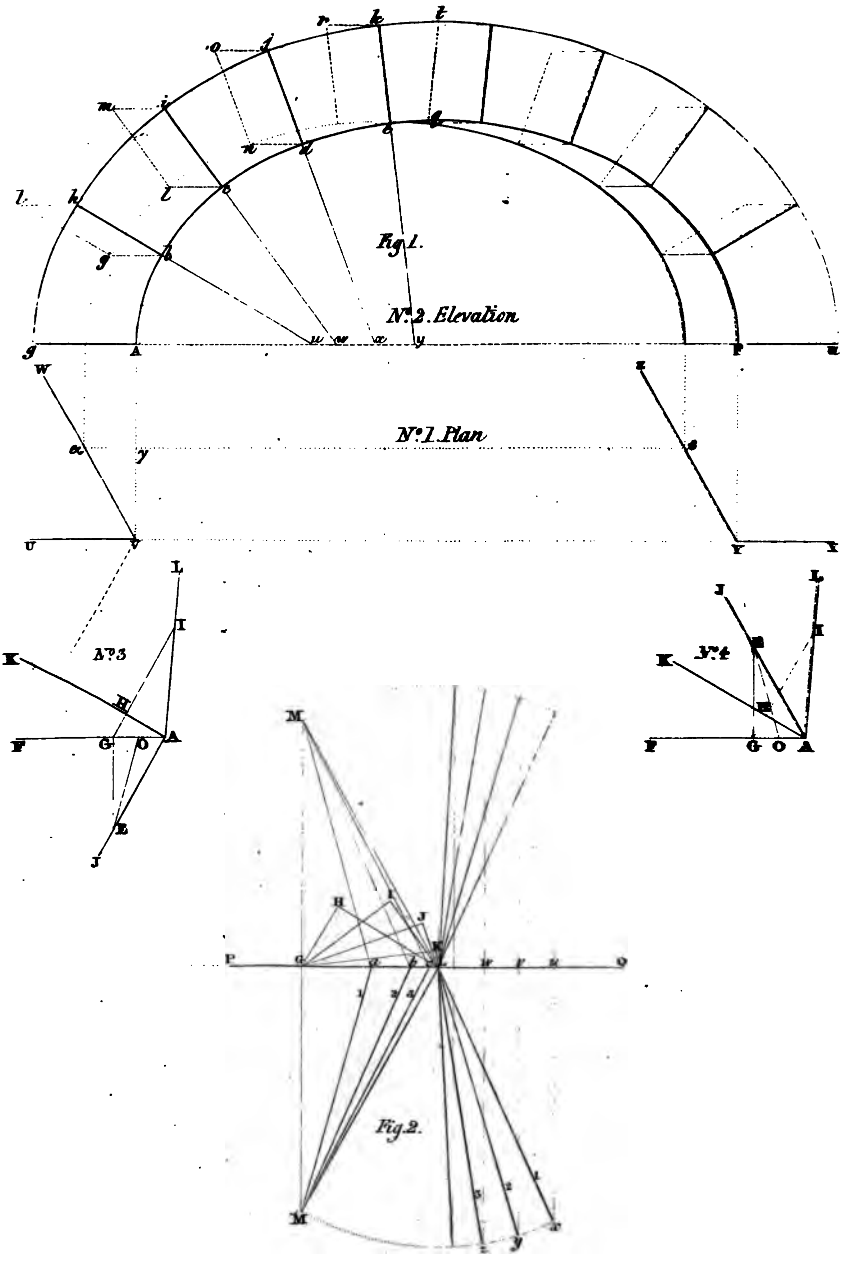


PLATE 15.

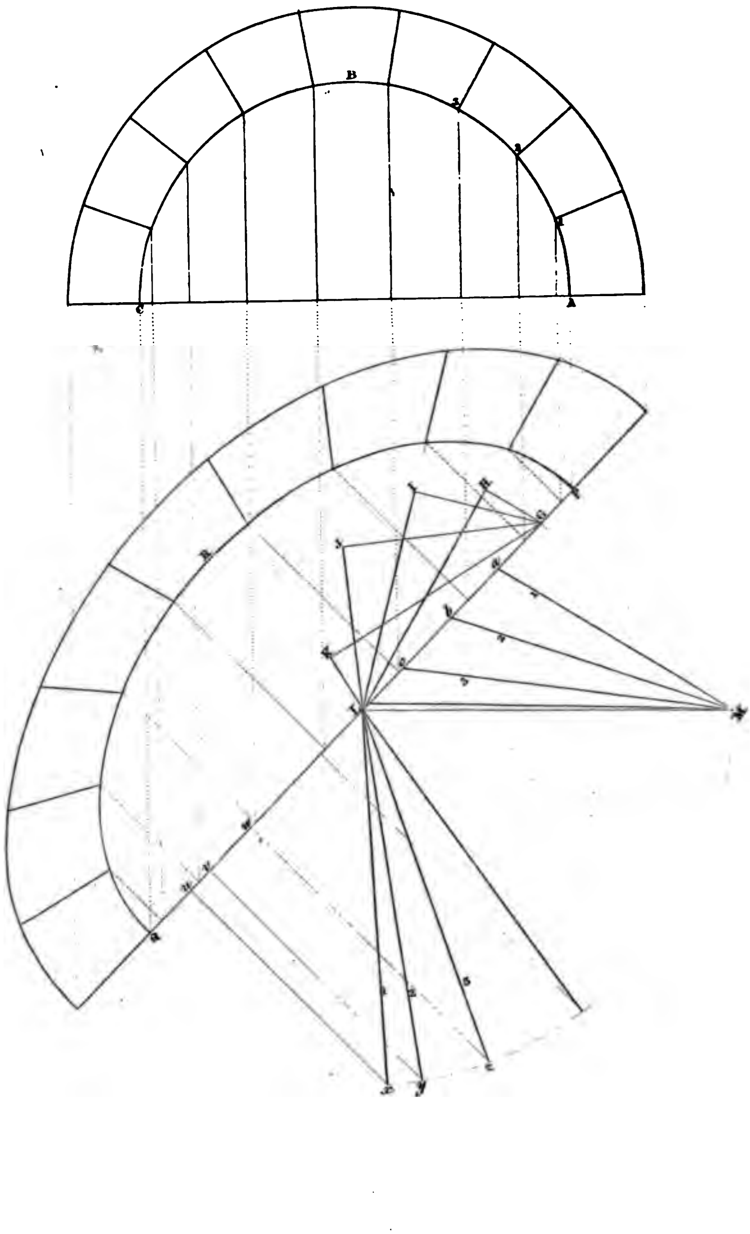
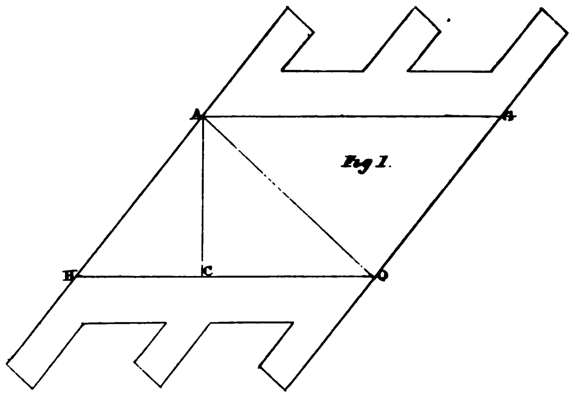
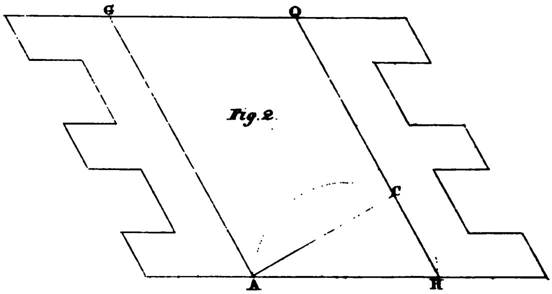
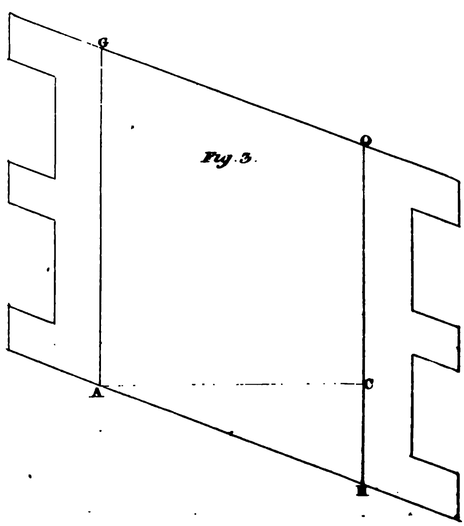


PLATE II.

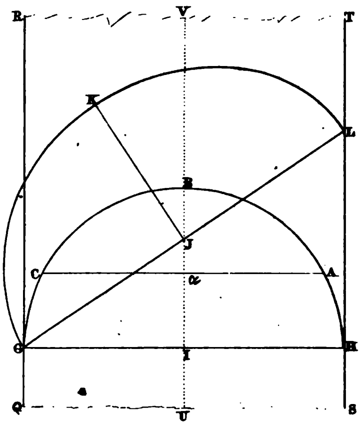
Introduction







Introduction



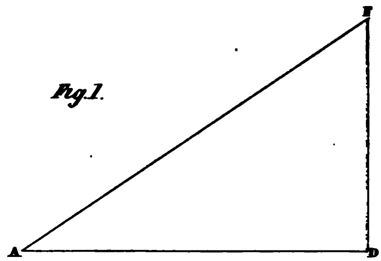
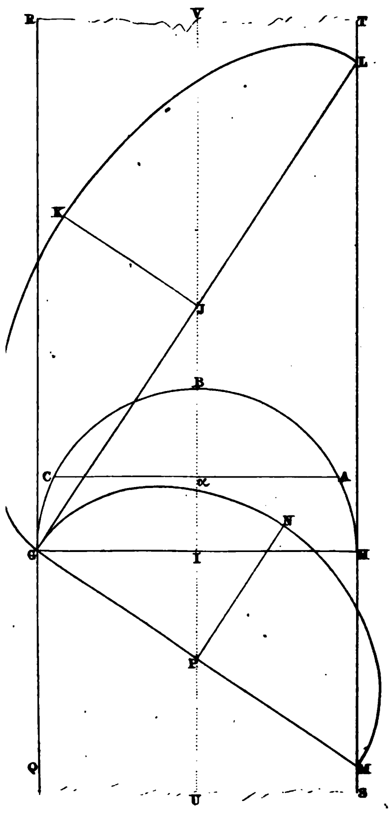


Fig. 1.



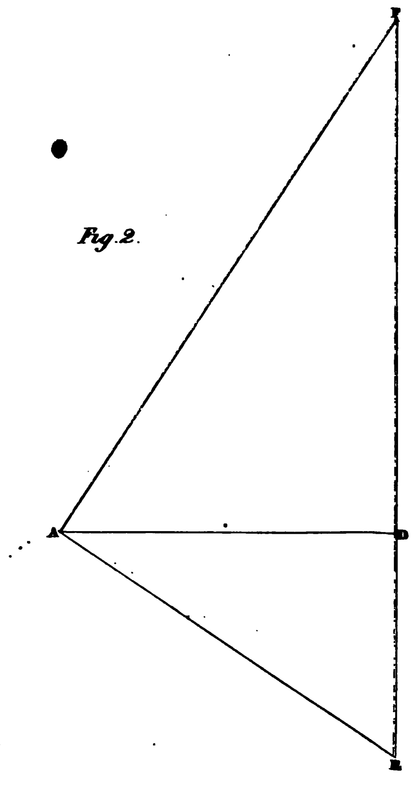


Fig. 2.

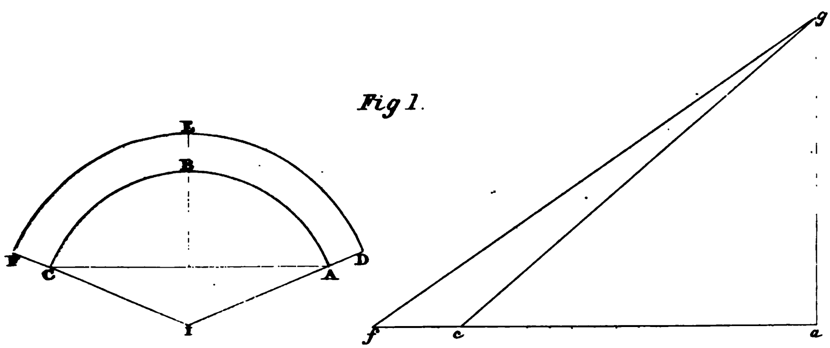


Fig 1.

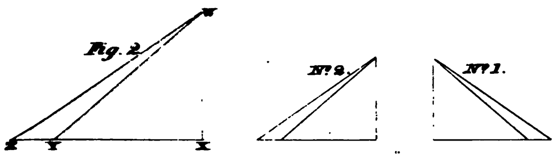


Fig. 2.

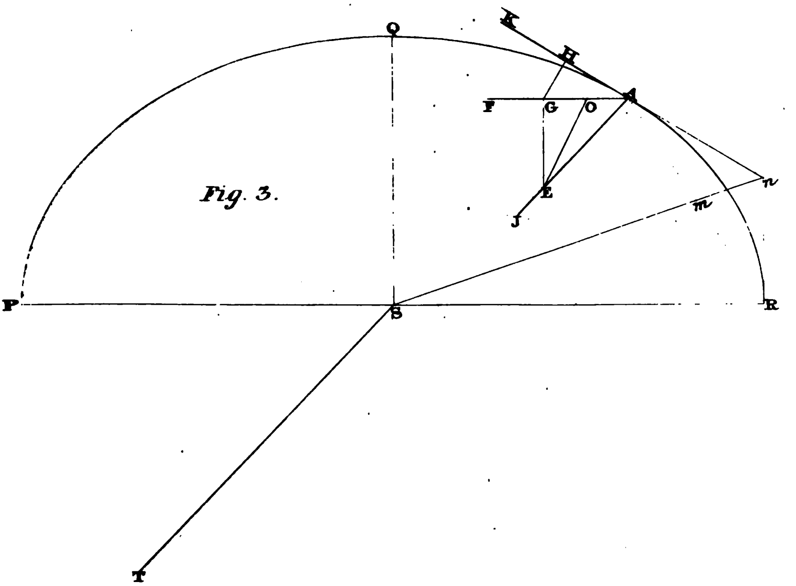


Fig. 3.

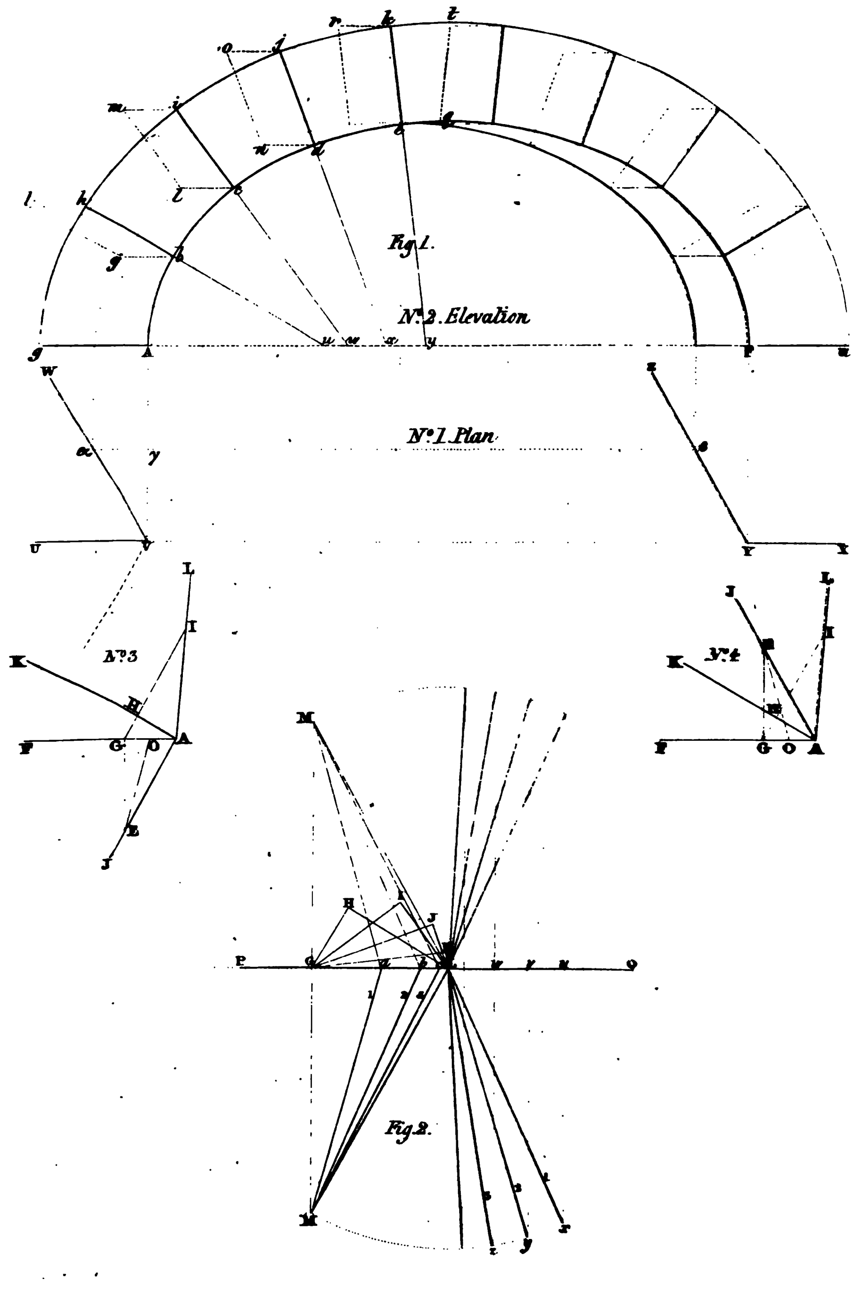


Fig. 1.
No. 2. Elevation
No. 1 Plan
No. 3
No. 4
Fig. 2.

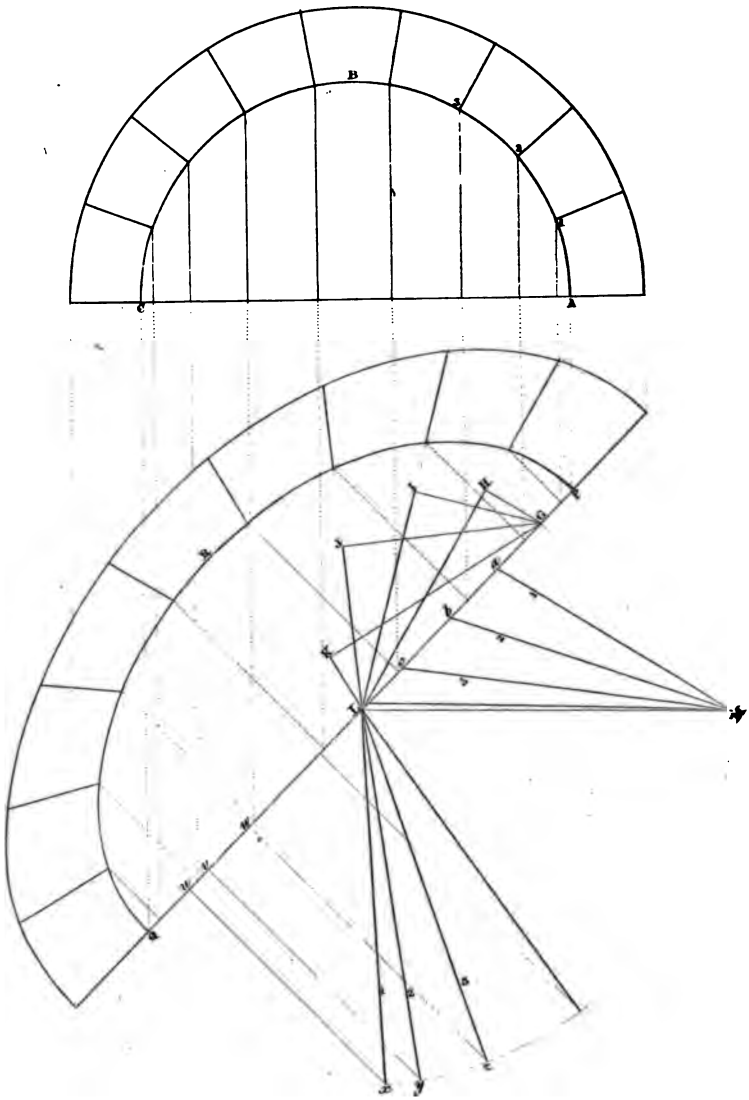

B
C
A
B
G
H
S
K
L
a
b
x
y
z
c

PLATE 16.

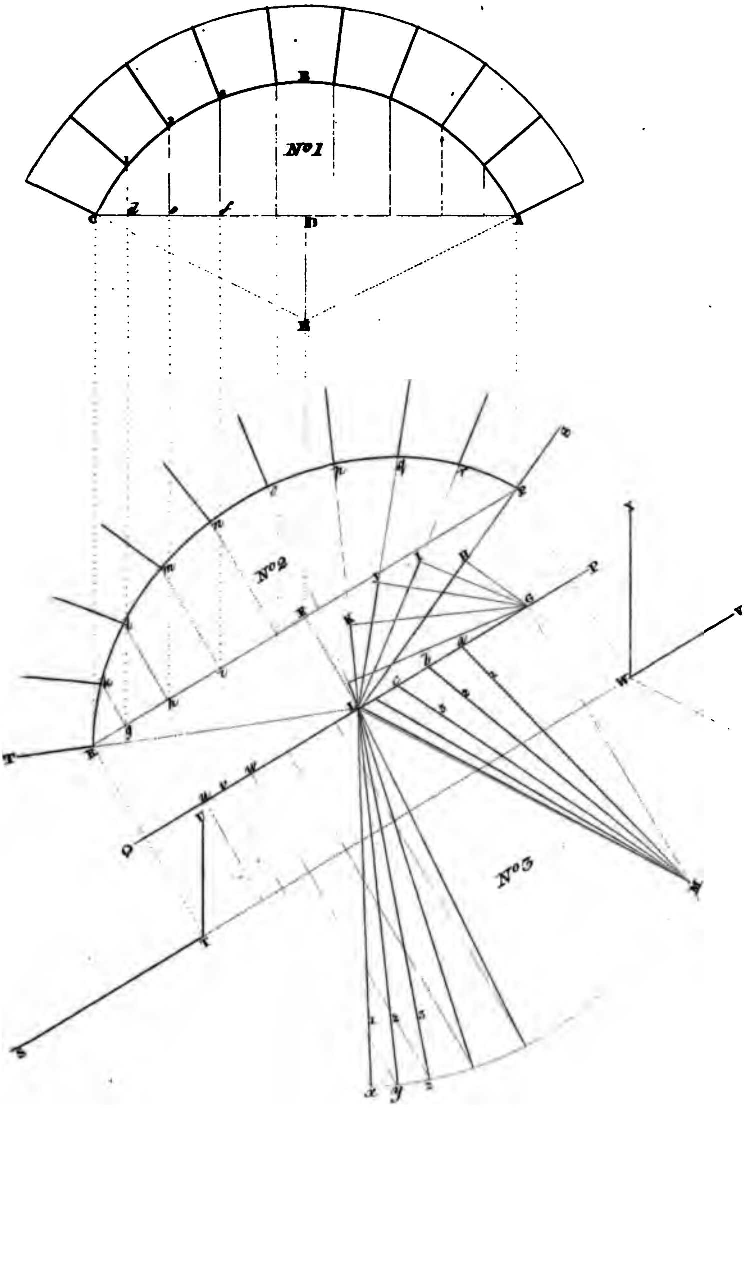
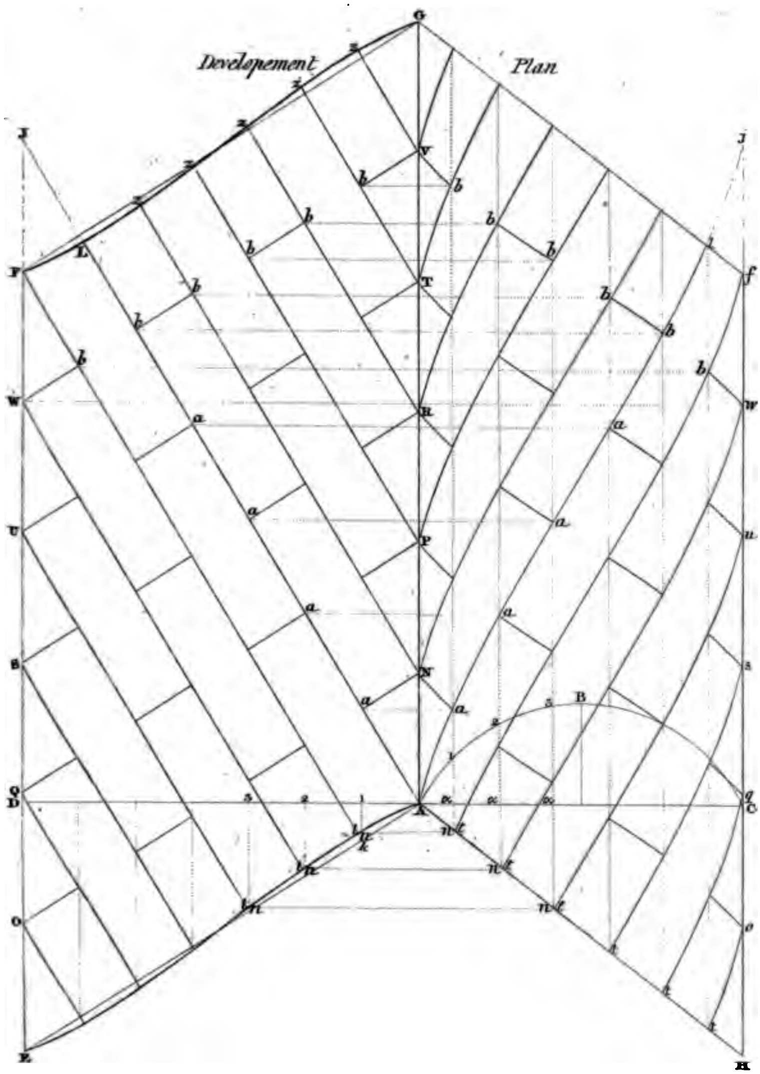


PLATE. 17.



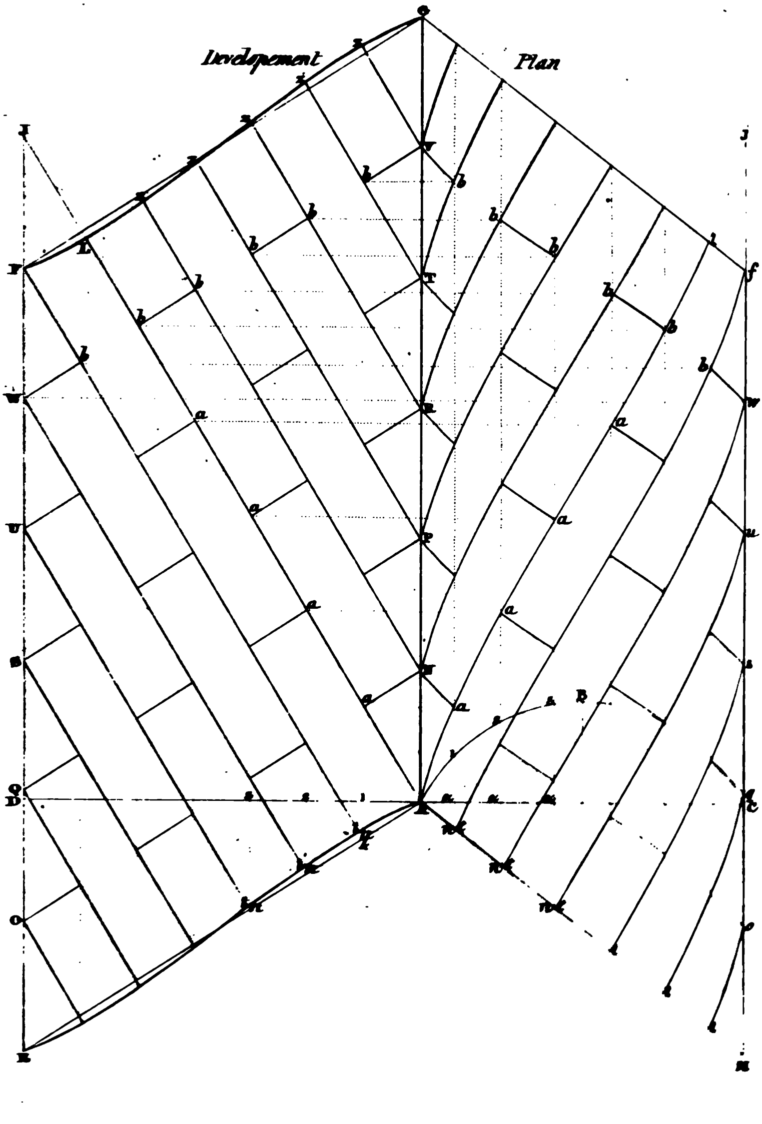

Developement
Plan

PLATE 20.

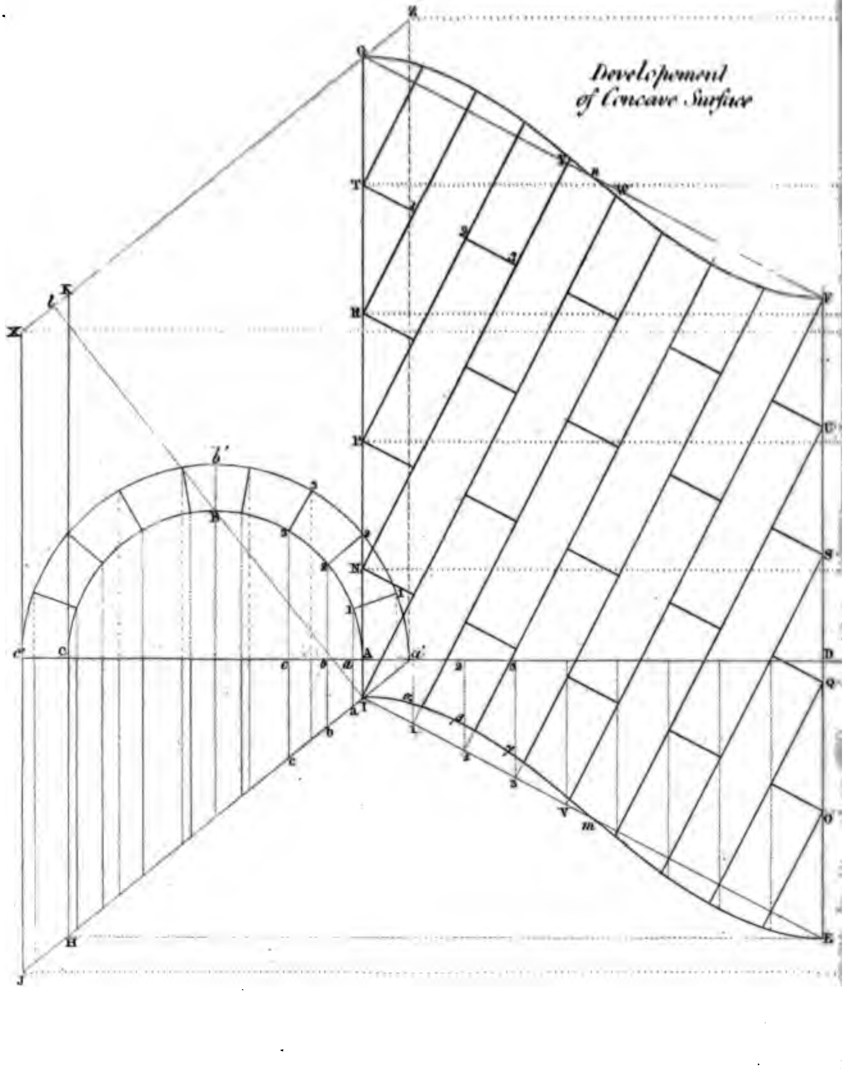


PLATE 21.

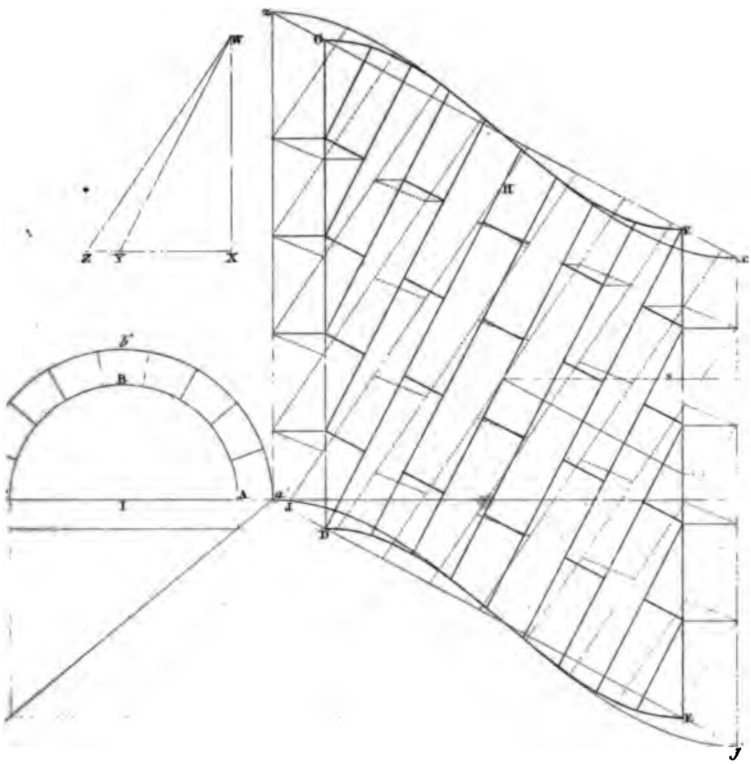


PLATE. 22.

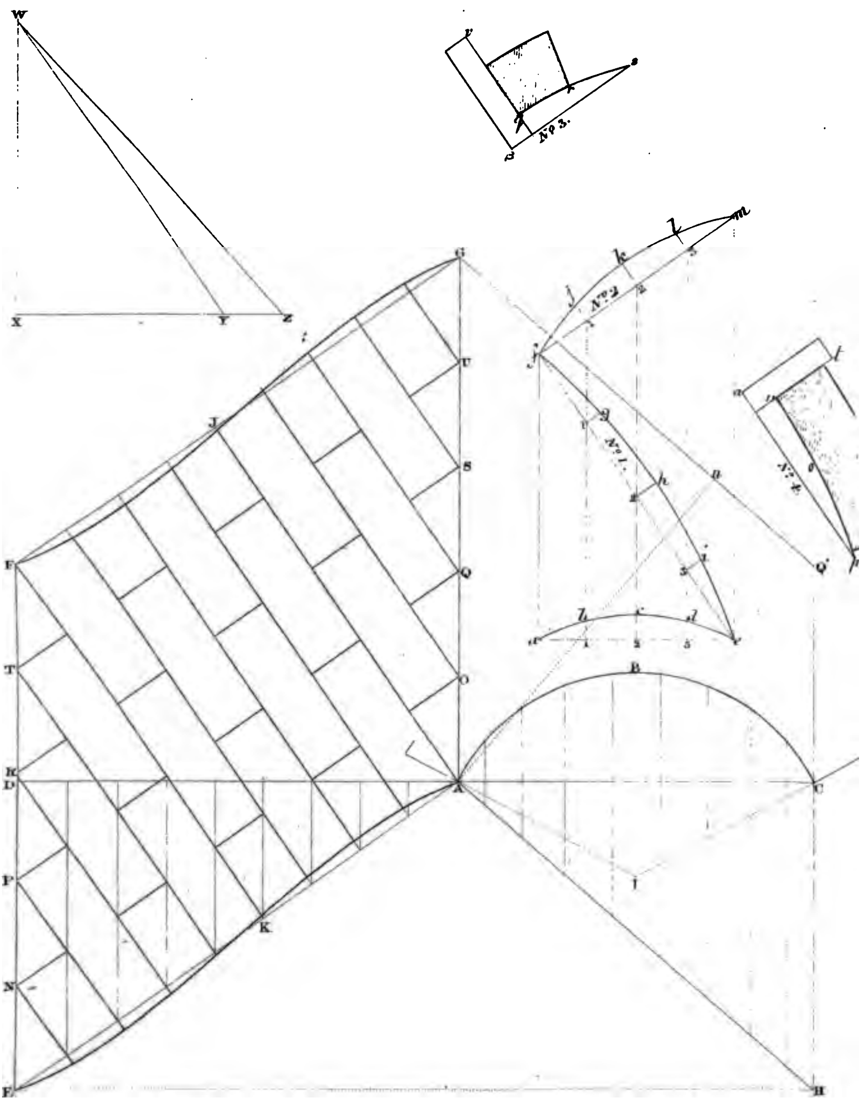


PLATE. 23.

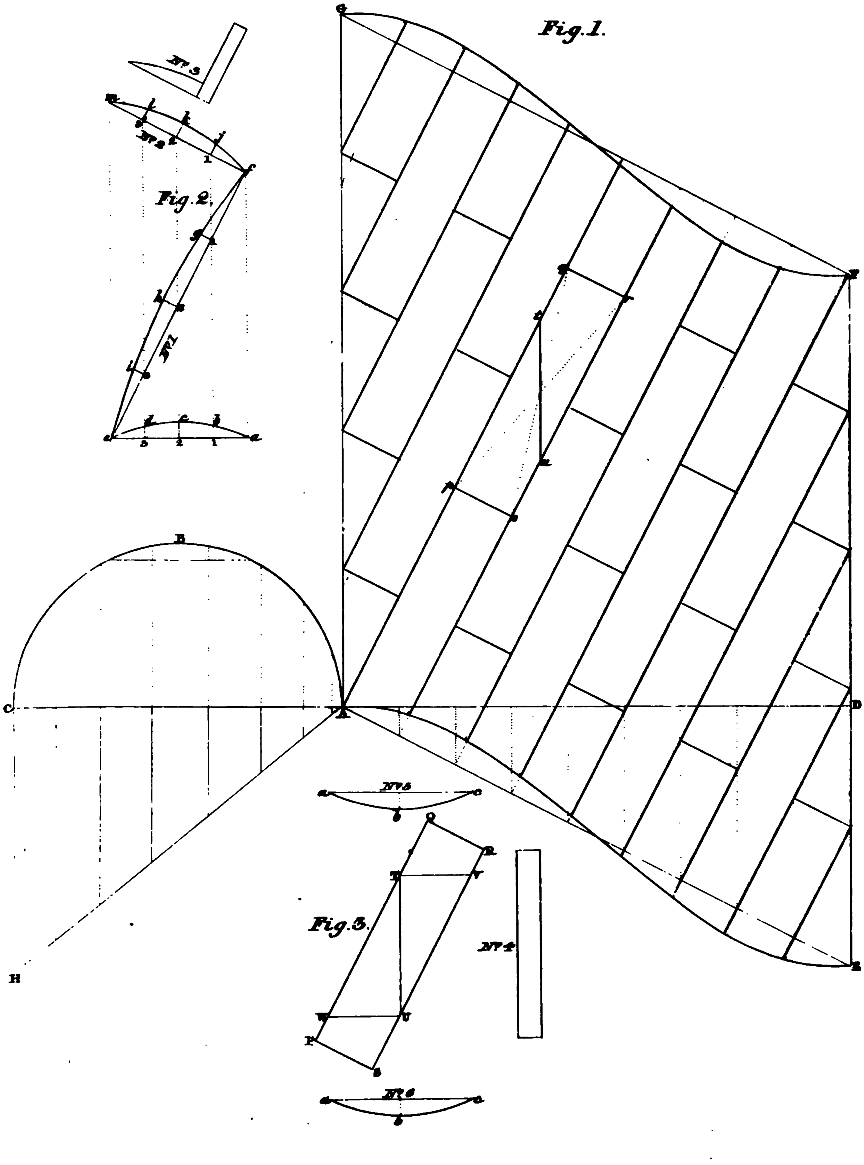


PLATE 24.

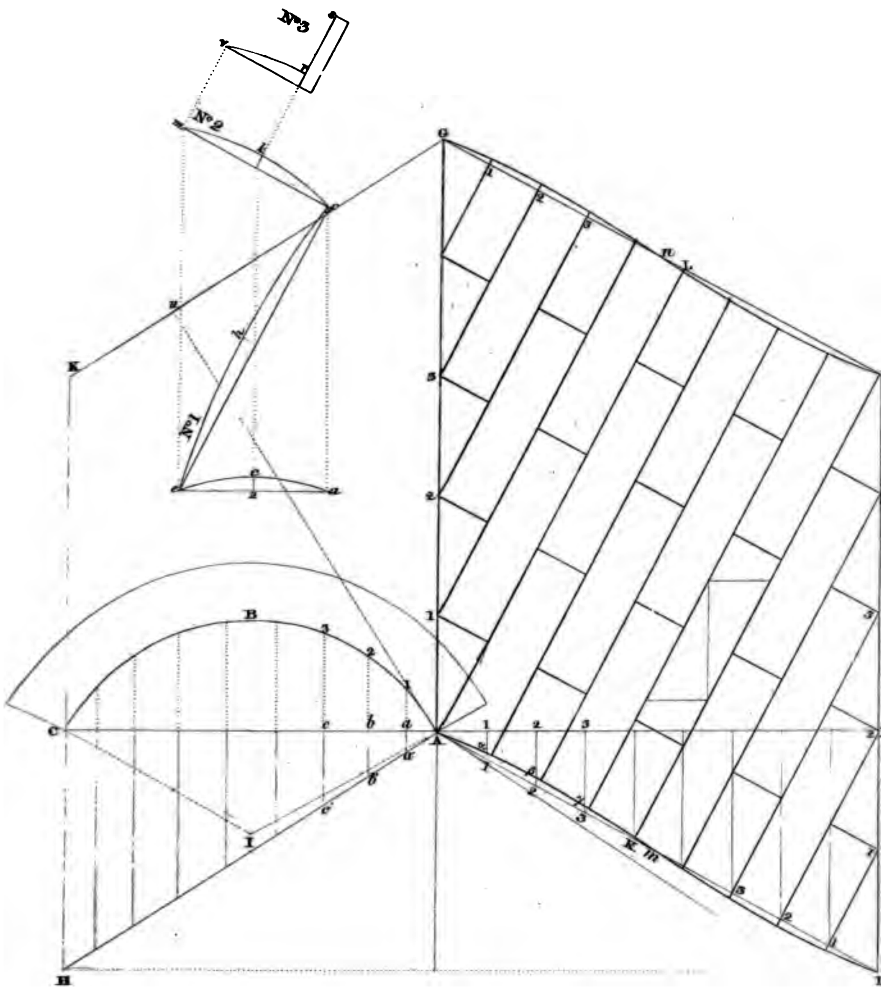
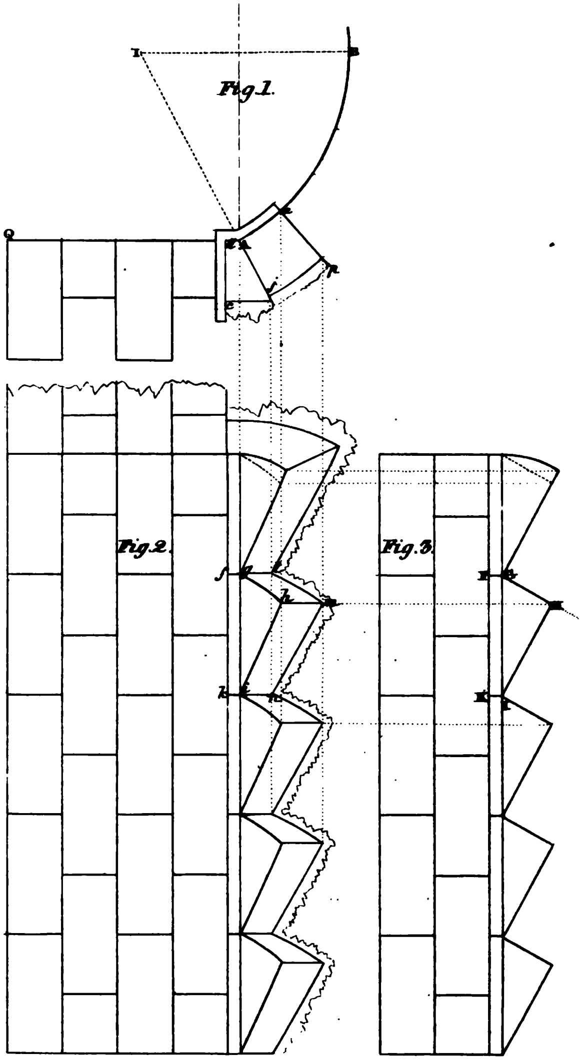
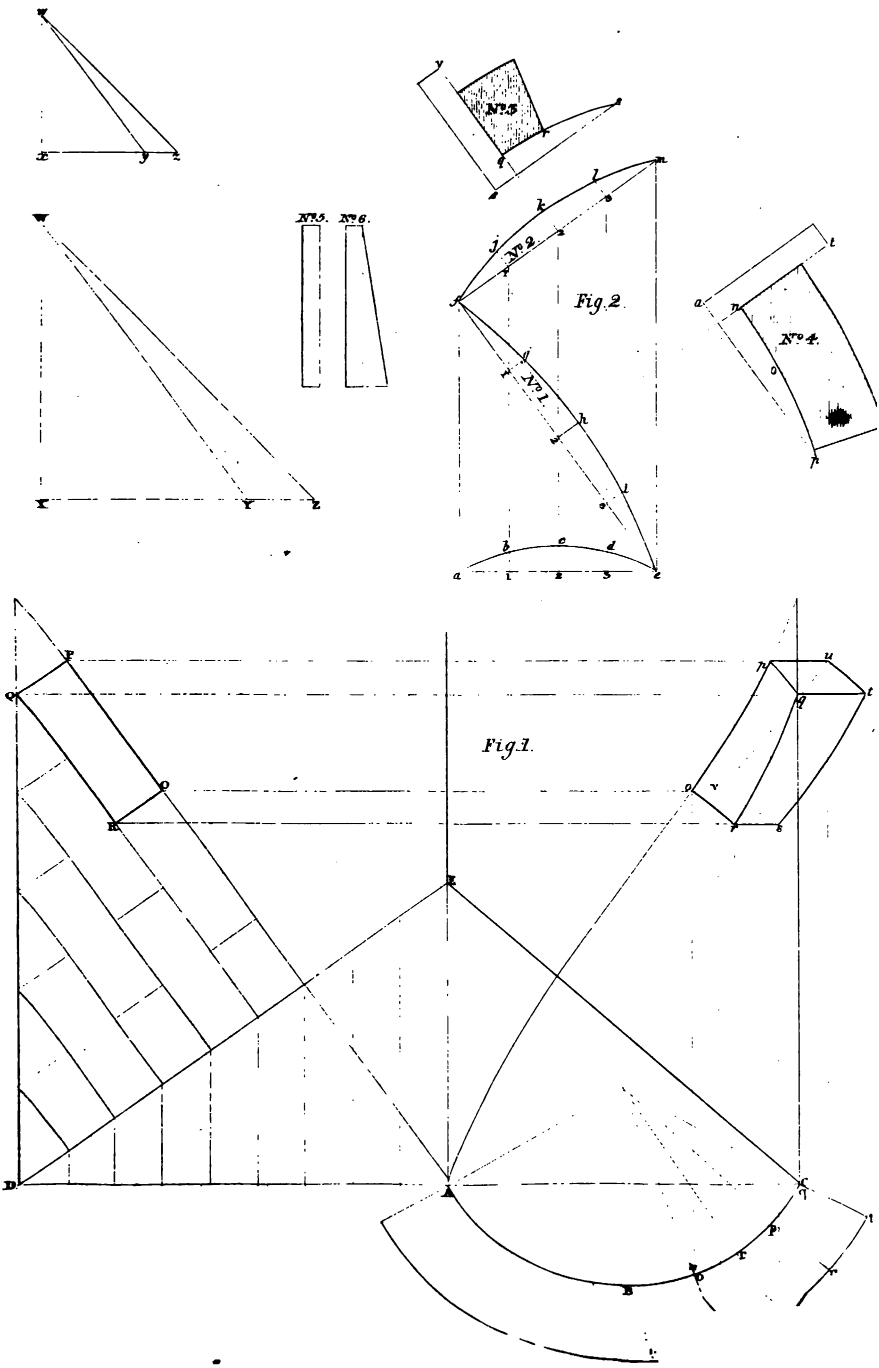


PLATE 25.




No. 1
No. 2
No. 3
No. 4
No. 5
No. 6
Fig. 2.
Fig. 1.

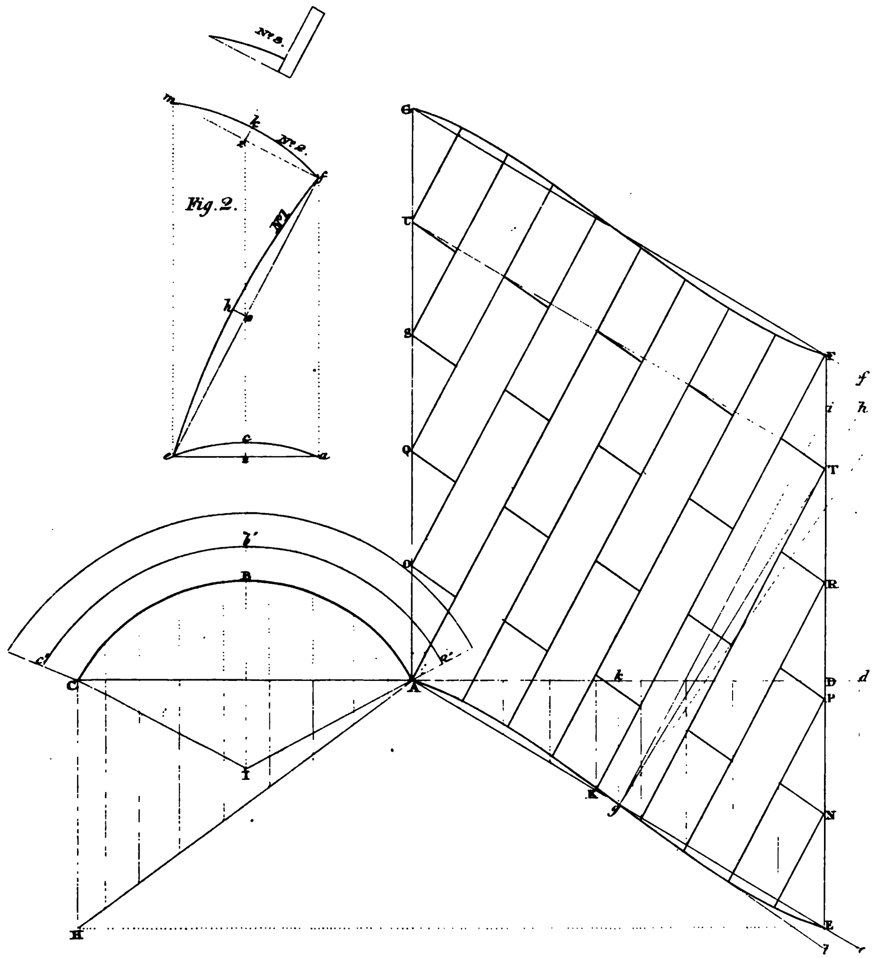

Nº 3.
Nº 2.
Fig. 2.
Nº 1.
m
k
f
h
e
c
a
G
U
S
Q
O
F
T
R
D
P
N
E
A
C
B
I
H
K
g
k
d

PLATE. 28.

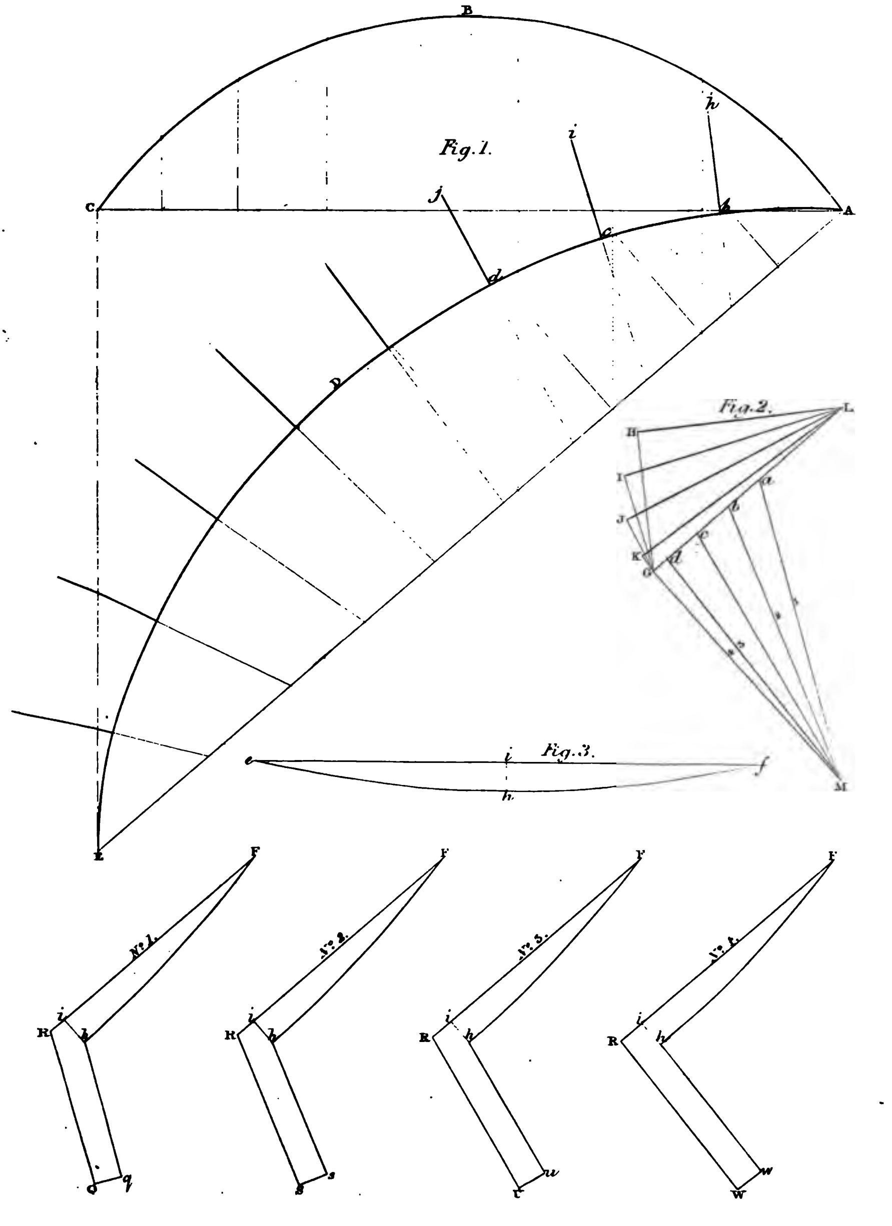


Fig. 1.
B
C
A
D
E
h
i
j
b
c
d
Fig. 2.
L
H
I
J
K
G
a
b
c
d
M
Fig. 3.
e
i
f
h
F
No. 1.
R
i
h
Q
q
No. 2.
S
s
No. 3.
T
u
No. 4.
W
w


v
s
k
h

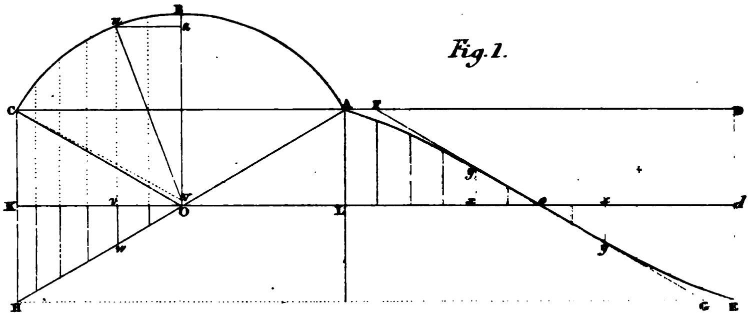


Fig. 1.

Fig. 2.

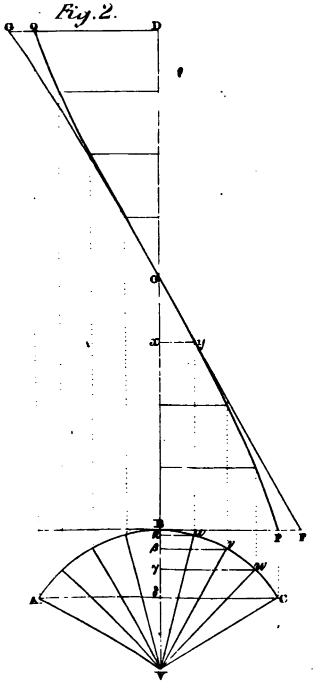
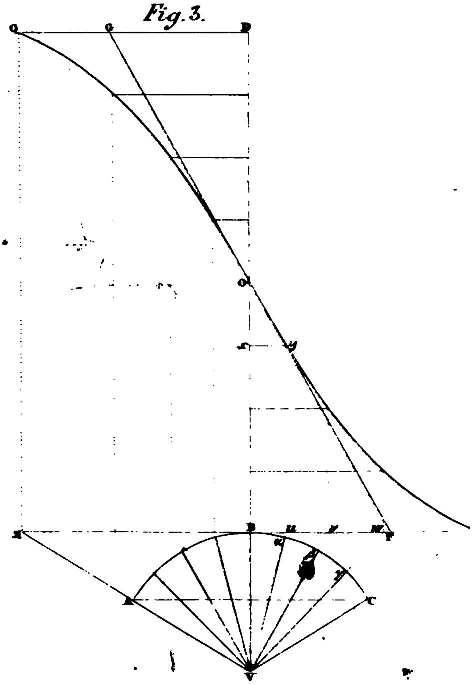


Fig. 3.



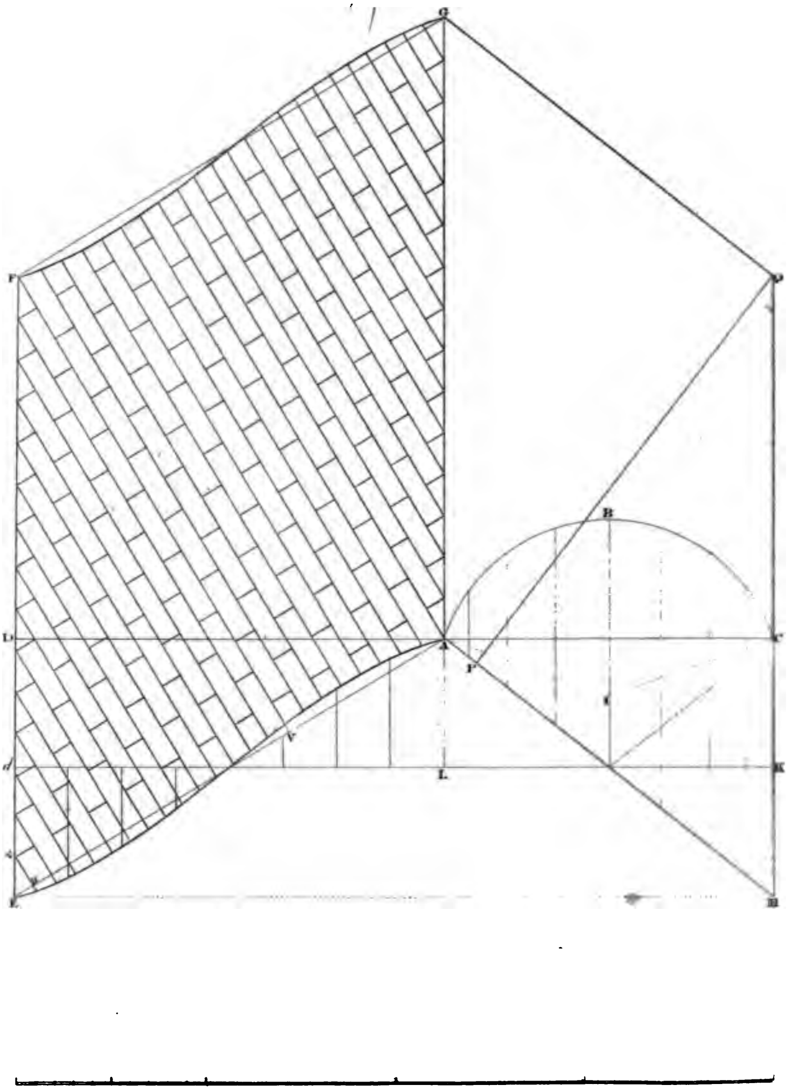

G
F
D
A
F
d
L
E
B
C
K
H

PLATE 32.

Fig.1

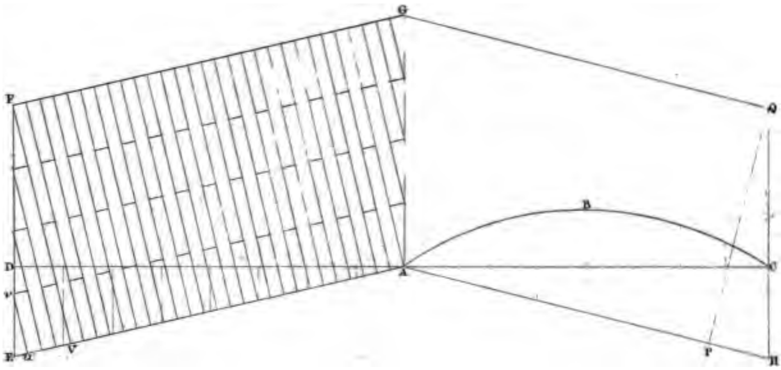


Fig.2

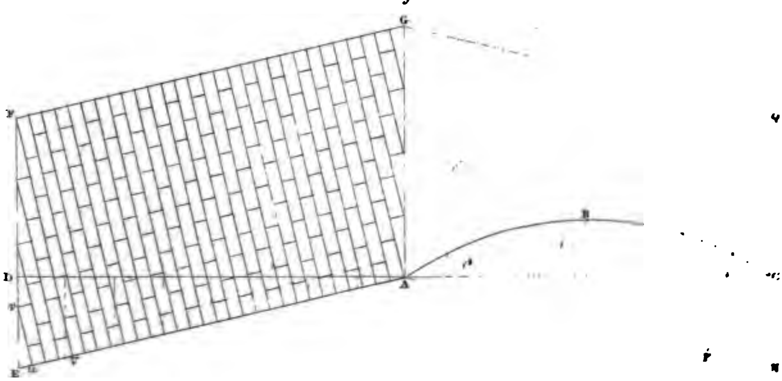


PLATE 33.

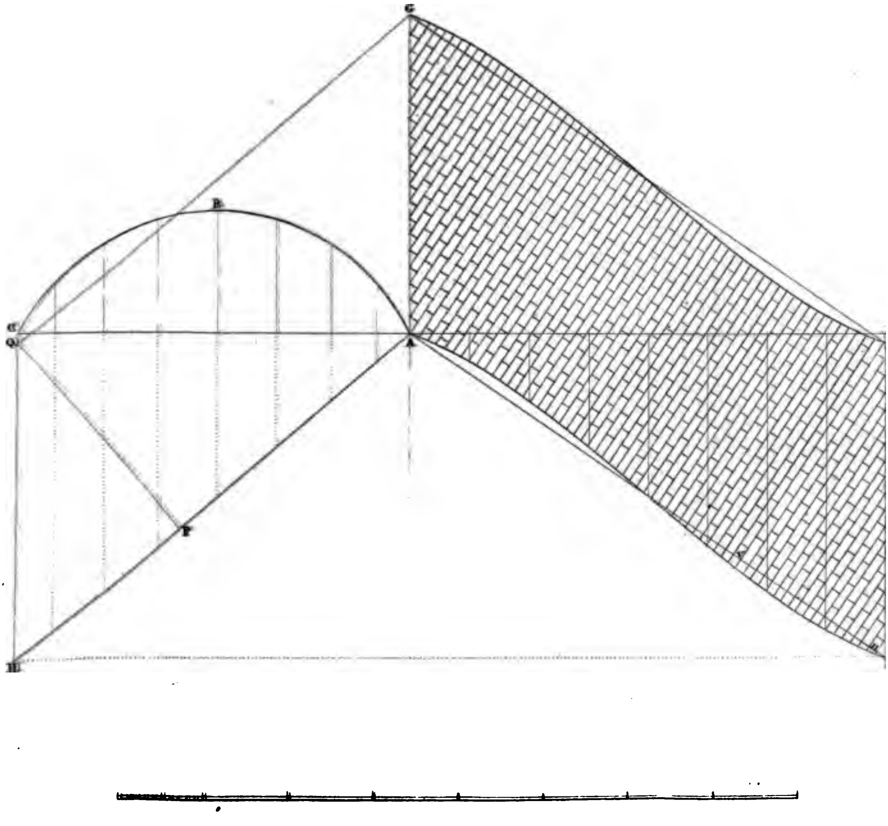


PLATE 34.

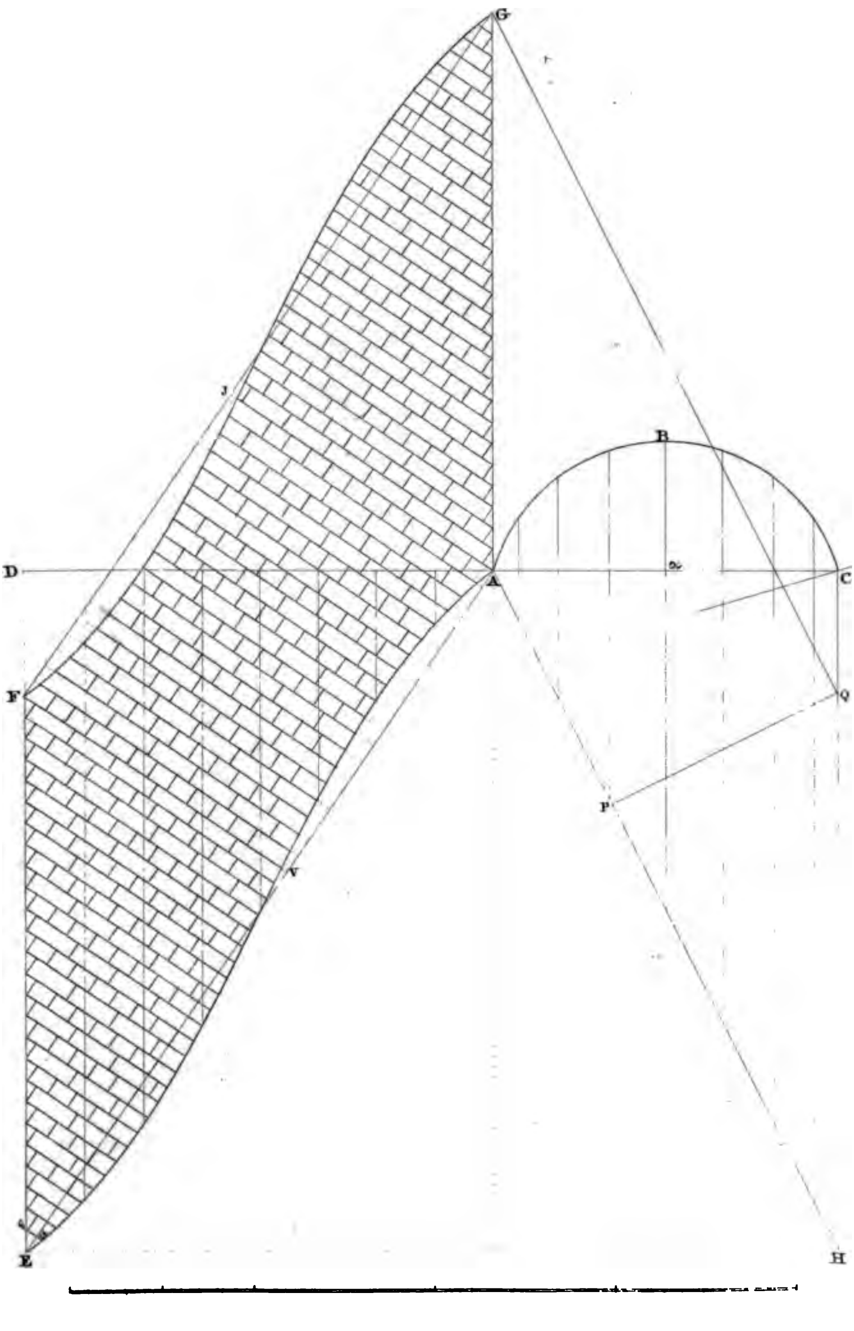


PLATE A

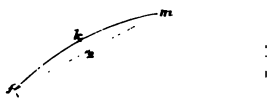


Fig. 2.

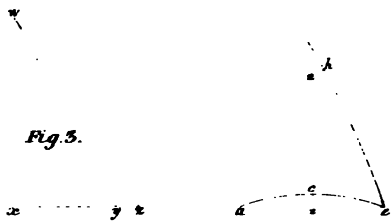


Fig. 3.

Fig. 1.

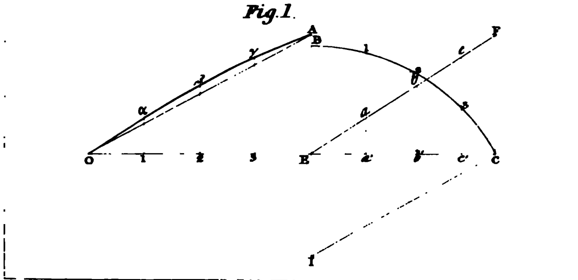


Fig. A.

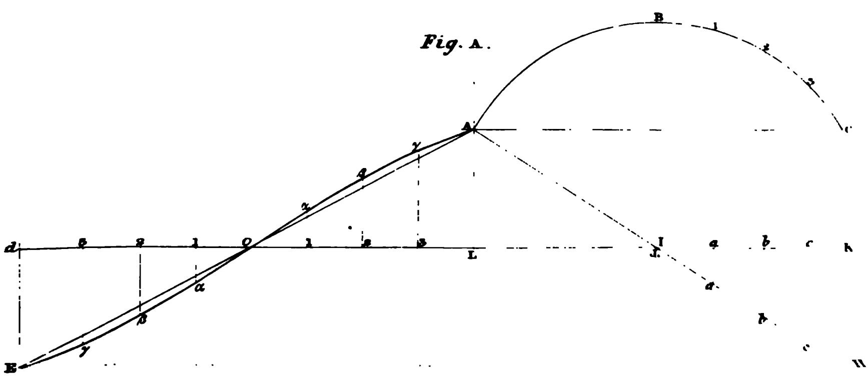


PLATE 35

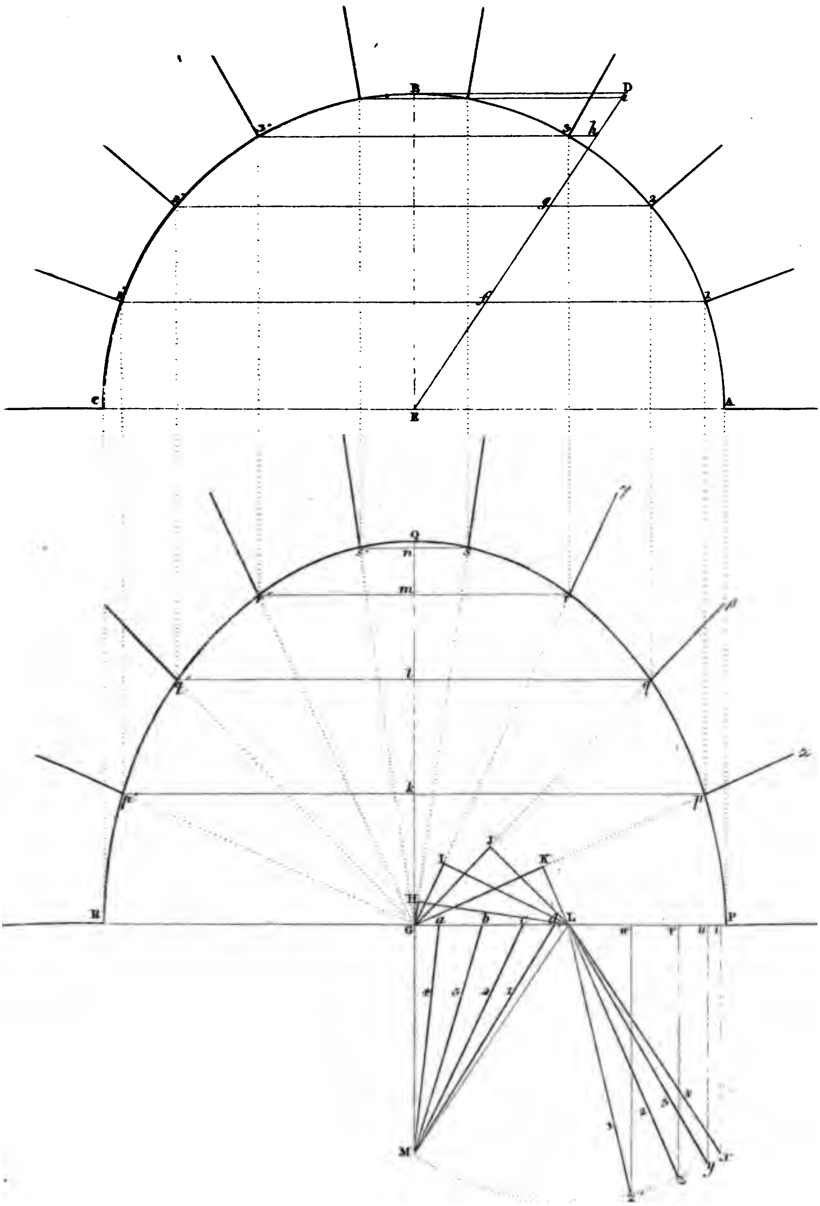


PLATE 36

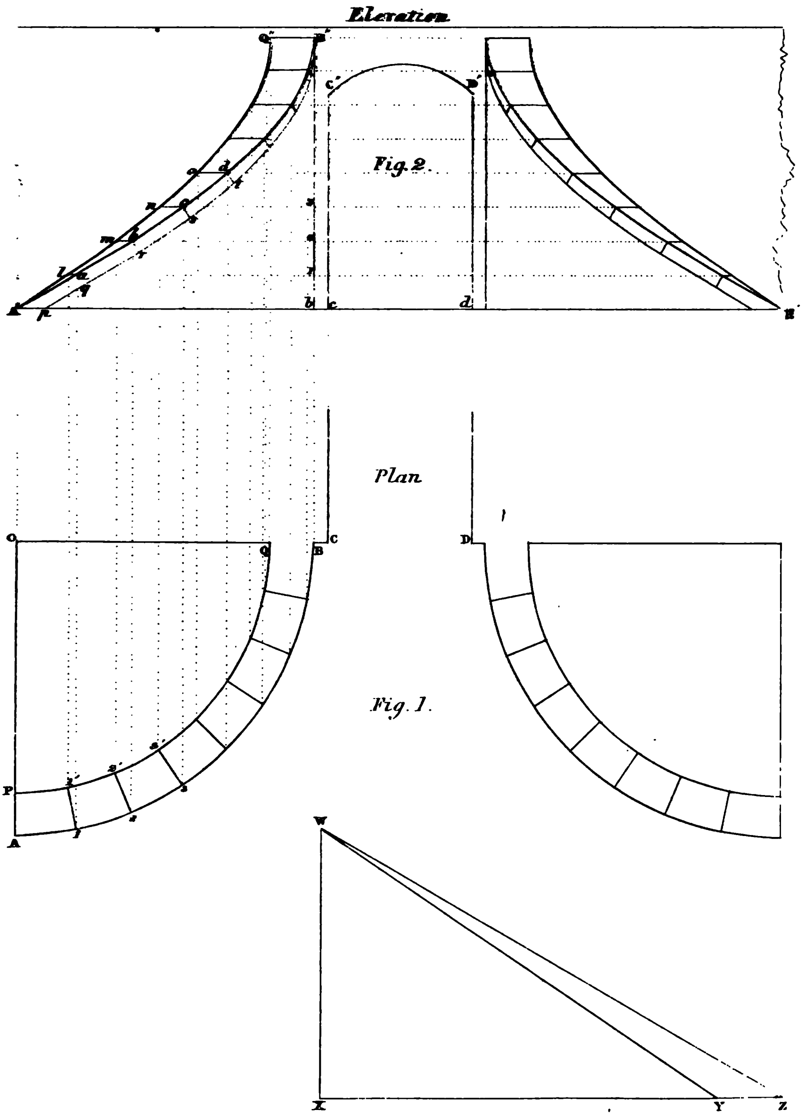
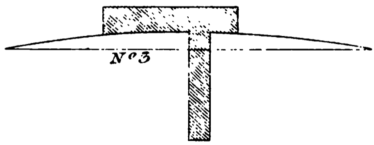


PLATE 37



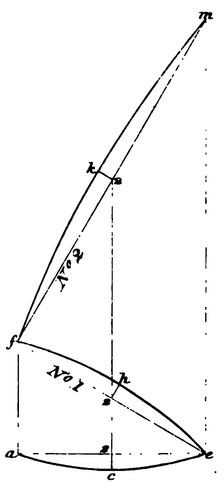
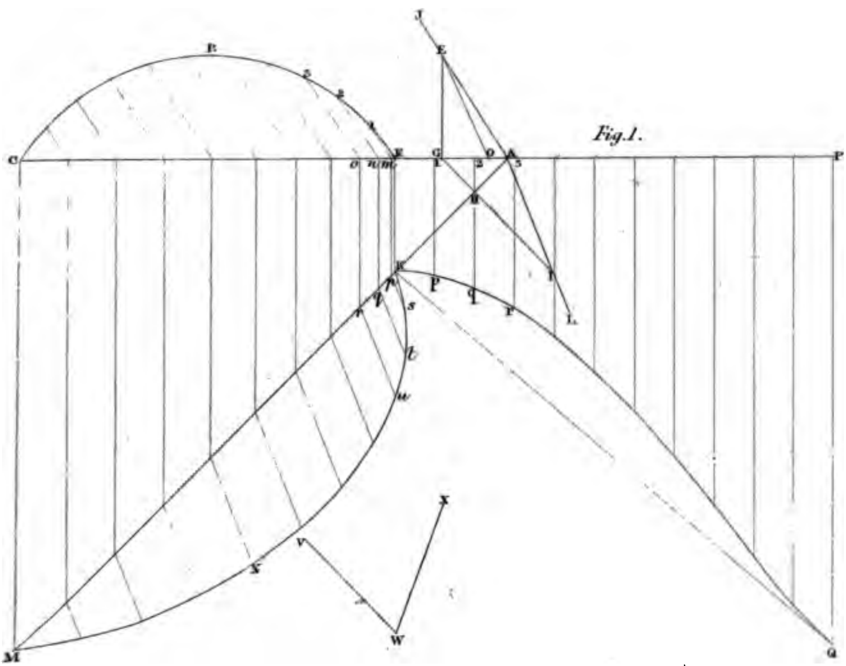


PLATE 38



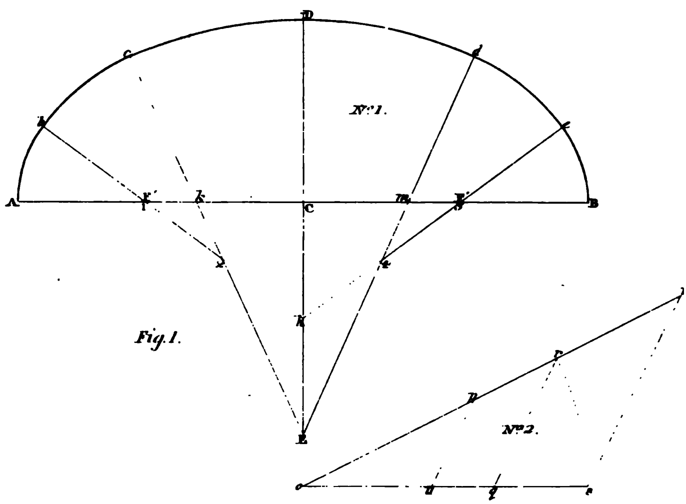


Fig. 1.

Fig. 2.